

# 3.3

## Order of Operations

### Focus on...

After this lesson, you will be able to...

- use the order of operations on expressions with powers
- apply the laws of exponents



In the game show *Power of 5*, contestants try to answer eight questions in their pursuit of \$10 million.

### Explore Order of Operations With Powers

1. How many times greater is each prize value than the previous prize value? Explain how you arrived at your answer.
2. a) What is an expression in exponential form that represents the prize value for answering the fourth question correctly? Compare your answer with a classmate's.  
b) How could you find the value of this expression?
3. Write expressions in exponential form for the top prize value and for the prize value for answering the fifth question correctly. Use these expressions to write an expression that shows the difference between these prize values. Then, evaluate the expression. Compare your answer with a classmate's.

### Reflect and Check

#### Literacy Link

A *coefficient* is a number that multiplies an expression. In  $-5(4)^2$ , the coefficient is  $-5$ .

4. a) Identify the coefficient and the power in the expression  $128 \times 5^7$ .  
b) What does each of these values represent in the *Power of 5* game show?
5. a) What does the expression  $128(5^7) - 128(5^3)$  represent in terms of prize values in the *Power of 5* game show?  
b) Describe the steps you would use to evaluate this expression.

The expression  $128 \times 5^7$  can also be written as  $128(5^7)$ .

## Link the Ideas

### Example 1: Determine the Product of a Power

Evaluate.

a)  $3(2)^4$       b)  $-3(-5)^2$       c)  $-4^4$

#### Solution

a) **Method 1: Use Repeated Multiplication**

You can use repeated multiplication for the power.

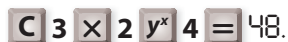
$$\begin{aligned} 3(2)^4 &= 3 \times 2^4 \\ &= 3 \times 2 \times 2 \times 2 \times 2 \\ &= 48 \end{aligned}$$

**Method 2: Use Order of Operations**

$$\begin{aligned} 3 \times 2^4 &= 3 \times 16 \\ &= 48 \end{aligned}$$

Evaluate the power first.

**Method 3: Use a Calculator**



$\boxed{3} \times \boxed{2} ^{\boxed{4}} = 48.$

b)  $-3(-5)^2 = -3(25)$   
 $= -75$

c)  $-4^4 = -1 \times 4^4$   
 $= -1 \times 256$   
 $= -256$



#### Literacy Link

The order of operations is:

- brackets
- exponents (powers)
- divide and multiply in order from left to right
- add and subtract in order from left to right

#### Tech Link

The key sequence may be different on different calculators. Experiment with performing order of operations on your calculator. Record the proper key sequence for your calculator.

#### Show You Know

Evaluate. Use a calculator to check your answer.

a)  $4 \times 3^2$

b)  $6(-2)^3$

c)  $-7^2$

## Example 2: Evaluate Expressions With Powers

Evaluate.

a)  $4^2 - 8 \div 2 + (-3^2)$       b)  $-2(-15 - 4^2) + 4(2 + 3)^3$

### Solution

a) *Method 1: Use Order of Operations*

$$\begin{aligned} & 4^2 - 8 \div 2 + (-3^2) \\ &= 16 - 8 \div 2 + (-9) \\ &= 16 - 4 + (-9) \\ &= 12 + (-9) \\ &= 3 \end{aligned}$$

*Method 2: Use a Calculator*

**C** **4** **y<sup>x</sup>** **2** **-** **8** **÷** **2** **+** **3** **+/-** **y<sup>x</sup>** **2** **=** 3.

You may need to use a different key sequence on your calculator.

b) *Method 1: Use Order of Operations*

$$\begin{aligned} & -2(-15 - 4^2) + 4(2 + 3)^3 \\ &= -2(-15 - 16) + 4(5)^3 \\ &= -2(-31) + 4(5)^3 \\ &= -2(-31) + 4(125) \\ &= 62 + 4(125) \\ &= 62 + 500 \\ &= 562 \end{aligned}$$

*Method 2: Use a Calculator*

**C** **2** **+/-** **x** **(** **15** **+/-** **-** **4** **x<sup>2</sup>** **)** **+** **4** **x** **(** **2** **+** **3** **)** **y<sup>x</sup>** **3** **=** 562.

### Show You Know

Evaluate.

a)  $4^2 + (-4^2)$       b)  $8(5 + 2)^2 - 12 \div 2^2$

### Key Ideas

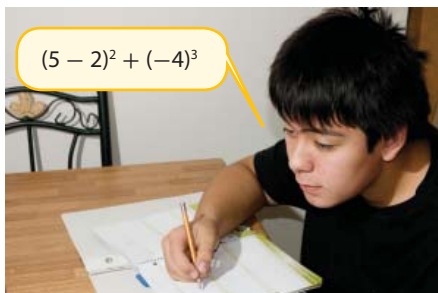
- Expressions with powers can have a numerical coefficient. Evaluate the power, and then multiply by the coefficient.
- Evaluate expressions with powers using the proper order of operations.
  - brackets
  - exponents
  - divide and multiply in order from left to right
  - add and subtract in order from left to right

Expression	Coefficient	Power	Repeated Multiplication	Value
$5(4^2)$	5	$4^2$	$5 \times 4 \times 4$	80
$(-2)^4$	1	$(-2)^4$	$(-2)(-2)(-2)(-2)$	16
$-3^4$	-1	$3^4$	$-1 \times 3 \times 3 \times 3 \times 3$	-81

## Check Your Understanding

### Communicate the Ideas

- Using the terms *coefficient* and *base*, explain why the two expressions  $-2^2$  and  $(-2)^2$  are different and result in different answers.
- Your classmate, Han, needs help with his homework. Explain how to evaluate  $(5 - 2)^2 + (-4)^3$ .



- Identify the incorrect step in the following solution. Show how to correct it. What is the correct answer?

$$\begin{aligned}(3 + 5)^2 - 4 \times 3^2 \\ &= 8^2 - 4 \times 3^2 && \text{Step 1} \\ &= 64 - 4 \times 3^2 && \text{Step 2} \\ &= 60 \times 3^2 && \text{Step 3} \\ &= 60 \times 9 && \text{Step 4} \\ &= 540 && \text{Step 5}\end{aligned}$$

- Maria was asked to evaluate  $128 \times 5^3$ . What mistake did Maria make in her solution?

$$\begin{aligned}128 \times 5^3 \\ &= 640^3 \\ &= 262\,144\,000\end{aligned}$$

## Practise

For help with #5 to #7, refer to Example 1 on page 109.

- Evaluate each expression.

a) $4(2)^5$	b) $7(-3)^2$
c) $-2(5^4)$	d) $3(-2^2)$

- Write each expression using a coefficient and a power. Then, find the value of each expression.

a) $4 \times 2 \times 2 \times 2 \times 2$
b) $3 \times (-2) \times (-2) \times (-2)$
c) $7(10)(10)(10)(10)(10)$
d) $-1 \times 9 \times 9 \times 9 \times 9$

- Write the key sequence you would use to evaluate each expression using your calculator. What is the answer?

a) $4 \times 3^2$
b) $-5(4)^3$

For help with #8 and #9, refer to Example 2 on page 110.

- Evaluate.

a) $3^2 + 3^2$
b) $(2 + 7)^2 - 11$
c) $7^3 - 3(-4)^3$
d) $9 + (-2)^3 - 2(-6^2)$

9. Find the value of each expression.

- a)  $7 - 2(3^2)$
- b)  $(-4 - 3)^2 + (-3)^2$
- c)  $(-2)^6 \div 4^3$
- d)  $24 - 2^2 + (7^2 - 5^2)$

**Apply**

10. For each pair of expressions, which one has a greater value? How much greater is it?

- a)  $3(2)^3$        $2(3)^2$
- b)  $(3 \times 4)^2$      $3^2 \times 4^2$
- c)  $6^3 + 6^3$        $(6 + 6)^3$

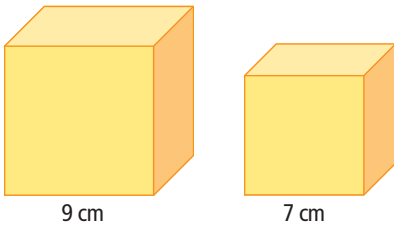
11. Find the step where Justin made an error. Show the correct answer.

$$\begin{aligned}
 &(-3 + 6)^2 - 4 \times 3^2 \\
 = &3^2 - 4 \times 3^2 && \text{Step 1} \\
 = &9 - 4 \times 9 && \text{Step 2} \\
 = &5 \times 9 && \text{Step 3} \\
 = &45 && \text{Step 4}
 \end{aligned}$$

12. Find the step where Katarina made an error. What is the correct answer?

$$\begin{aligned}
 &32 \div (-2)^3 + 5(4)^2 \\
 = &32 \div (-8) + 5 \times 8 && \text{Step 1} \\
 = &-4 + 5 \times 8 && \text{Step 2} \\
 = &-4 + 40 && \text{Step 3} \\
 = &36 && \text{Step 4}
 \end{aligned}$$

13. Write an expression with powers to determine the difference between the volume of the small cube and the volume of the large cube. What is the difference?



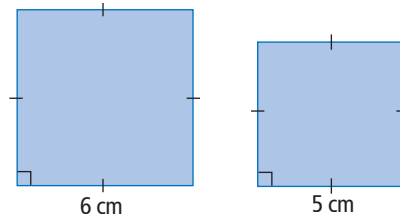
14. Read the following riddle and then answer the questions below.

In downtown Victoria, there are seven pink houses. Every pink house has seven pink rooms, every pink room has seven cats, and every cat has seven kittens.

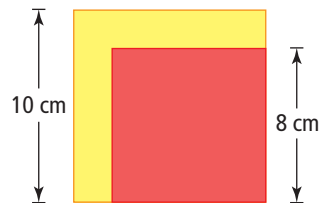
- a) How many pink rooms are there?
- b) How many kittens are there?
- c) Write an expression using powers of 7 to determine the total number of houses, rooms, cats, and kittens. Evaluate your expression.



15. Write an expression with powers to determine the difference between the area of the large square and the area of the small square. What is the difference?



16. A red square with a side length of 8 cm is placed on a yellow square with a side length of 10 cm. Write an expression with powers to determine the visible yellow area. What is the visible yellow area?



## Extend

17. What is the value of  $5^{3^2}$ ?
18. In a game show called The Pyramid of Money, a contestant must successfully answer a set of questions. The first question is worth \$3125. Each question after that is worth four times the value of the previous question.
- What is the value of question 2? question 3? question 4?
  - How many questions would a contestant have to answer correctly before becoming eligible to answer a question with a value of \$3 200 000?
  - Which question is represented by the expression  $3125 \times 4^7$ ?
  - Write an expression with powers that represents the sum of the values of the first four questions.
19. A phone tree is used to notify the players on a football team about a change in the time for their next game. Each of the three coaches calls two different players, and then each player calls two more players. Each person only makes two calls. The chart shows the number of people calling and receiving calls for each round of two calls.

Round of Calls	Number of People Calling	Number of Calls Received	Total Number of People Notified
First	3	6	$3 + 6 = 9$
Second	6	12	$9 + 12 = 21$
Third	12	24	$21 + 24 = 45$
Fourth	24	■	$45 + \blacksquare = 93$

- What value belongs in each unknown box?
  - Write an expression for the number from part a) as a product of 3 and a power of 2.
  - What does the 3 in part b) represent?
  - What does the exponent in part b) represent?
  - Imagine that the phoning continued. Determine an expression for the number of calls received in the sixth round and evaluate it.
  - If five coaches started the phone tree instead of three, what would be the number of calls received in the third round?
20. Use four 2s to write an expression with the greatest possible value.

## Math Link

You are planning to build a mobile with a cylinder and a cube.

- The height and radius of the cylinder and the height of the cube will all be the same measurement. Choose a whole number measurement, in centimetres.
- Write an expression in exponential form to calculate the difference in the area of material required to make each shape. Which shape requires more material? How much more? Express your answer to the nearest tenth of a square centimetre.
- Write an expression in exponential form to calculate the total area of material needed to make both shapes. Express your answer to the nearest tenth of a square centimetre.

