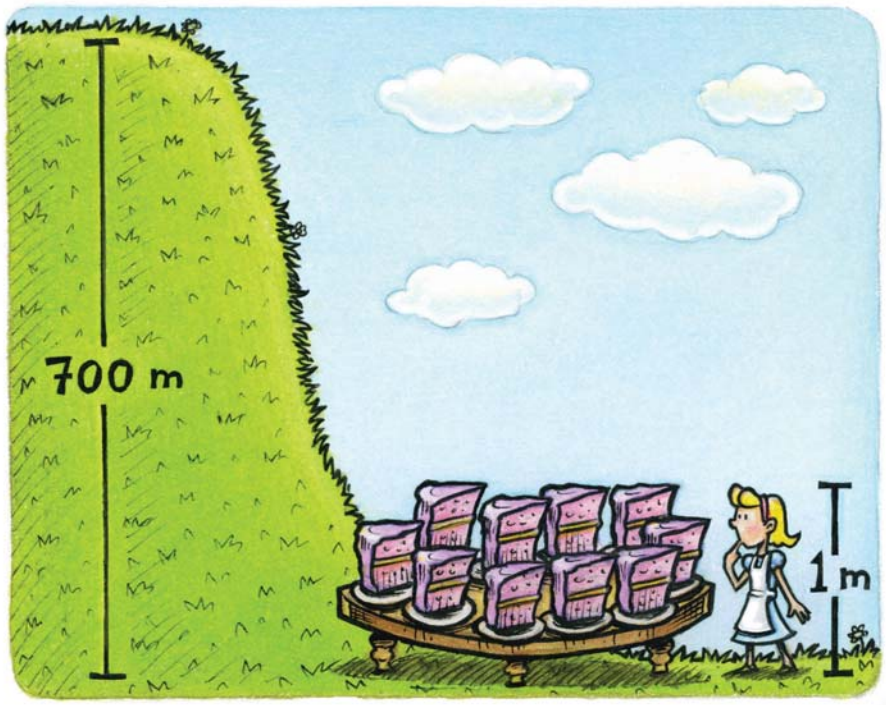
3.1 Using Exponents to Describe Numbers

Focus on...

After this lesson, you will be able to...

- represent repeated multiplication with exponents
- describe how powers represent repeated multiplication



In the story *Alice in Wonderland*, Alice could change her size dramatically by eating cake. If she needed to triple her height, she would eat a piece of cake. Imagine that she is currently 1 m tall. She needs to increase her height to 700 m in order to see over a hill. How many pieces of cake do you think she will need to eat?

Materials

- calculator

Explore Repeated Multiplication

1. Create a table that shows how Alice's height changes after eating one, two, and three pieces of cake. Describe any patterns you see in the table.

2. a) How many pieces of cake does Alice need to eat to become at least 700 m tall? Show how you arrived at your answer.

b) What is Alice's height after eating the number of pieces of cake in part a)?

c) How many factors of 3 do you need to multiply to obtain your answer to part b)?

3. Explore how you could use a calculator to determine Alice's height after eating eight pieces of cake. Share your method(s) with your classmates. Record the methods that work for your calculator.

Reflect and Check

4. a) The expression 3^2 can be used to represent Alice's height after eating two pieces of cake. What does this expression mean in terms of factors of 3?

b) How could you represent $3 \times 3 \times 3 \times 3 \times 3$ as a **power**? Identify the **base** and **exponent**.

5. What is Alice's height after eating ten pieces of cake?

Link the Ideas

Example 1: Write and Evaluate Powers

a) Write $2 \times 2 \times 2 \times 2 \times 2$ in **exponential form**.
b) Evaluate the power.

Solution

a) There are five factors of 2 in the expression $2 \times 2 \times 2 \times 2 \times 2$.
$2 \times 2 \times 2 \times 2 \times 2$ can be written as the power 2^5.
The base of the power is 2 and the exponent of the power is 5.

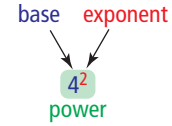
b) The product $2 \times 2 \times 2 \times 2 \times 2$ is 32.
So, $2^5 = 32$.

Show You Know

a) Write $4 \times 4 \times 4$ as a power.
b) Evaluate the power.

power

- an expression made up of a base and an exponent



base

- the number you multiply by itself in a power

exponent

- the number of times you multiply the base in a power

exponential form

- a shorter way of writing repeated multiplication, using a base and an exponent
- $5 \times 5 \times 5$ in exponential form is 5^3

Literacy Link

There are several ways to read powers.

You can read 2^5 in the following ways:

- two to the fifth
- two to the exponent five

History Link

Euclid was a Greek mathematician who lived from about 325 BCE to about 265 BCE. He was the first person to use the term *power*. He referred to power only in relation to squares.

The first time that the term *power* was used to refer to expressions with exponents greater than 2 was in 1696 in *Arithmetic* by Samuel Jeake.

Euclid

Example 2: Powers With Positive Bases

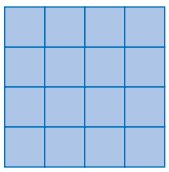
Evaluate each power.

a) 4^2 **b)** 2^3 **c)** 3^6

Solution

Strategies
Model It

a) The power 4^2 can be read as "four squared."
You can use a model of a square to represent the power.



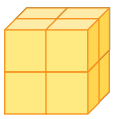
How does the model of the square represent 4^2?

Each side of the square is 4 units in length.
The area of the square is 16 because there are 16 small squares altogether in the square.

In the power 4^2, the base is 4 and the exponent is 2.

$$4^2 = 4 \times 4$$
$$= 16$$

b) The power 2^3 can be read as "two cubed."
You can use a model of a cube to represent the power.



How does the model of the cube represent 2^3?

Each edge of the large cube is 2 units in length.
The volume of the large cube is 8 because there are 8 samll cubes altogether in the large cube.

In the power 2^3, the base is 2 and the exponent is 3.

$$2^3 = 2 \times 2 \times 2$$
$$= 8$$

c) In the power 3^6, the base is 3 and the exponent is 6. You can represent 3^6 as repeated multiplication.

$$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$$
$$= 729$$

You could think of 3^6 as

$(3 \times 3) \times (3 \times 3) \times (3 \times 3)$
$= 9 \times 9 \times 9$
$= 9^3$

or

$(3 \times 3 \times 3) \times (3 \times 3 \times 3)$
$= 27 \times 27$
$= 27^2$

Are there other possibilities?

Show You Know

Evaluate each power.

a) 6^2 **b)** 3^4 **c)** 5^3

Example 3: Powers With Negative Bases

Evaluate each power.

a) $(-2)^4$

b) -2^4

c) $(-4)^3$

d) $-(-5)^6$

Solution

a) In the power $(-2)^4$, the base is -2 and the exponent is 4.
The exponent applies to the negative sign because -2 is enclosed in parentheses.

You can write the power as repeated multiplication.

$$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2)$$
$$= 16$$

Why is the answer positive?

b) In the power -2^4, the base is 2 and the exponent is 4.
The exponent does not apply to the negative sign because -2^4 is the same as $-(2^4)$.

$$-2^4 = -(2^4)$$
$$= -(2 \times 2 \times 2 \times 2)$$
$$= -16$$

c) In the power $(-4)^3$, the base is -4 and the exponent is 3.

$$(-4)^3 = (-4) \times (-4) \times (-4)$$
$$= -64$$

Why is the answer negative?

d) In the expression $-(-5)^6$, the base is -5 and the exponent is 6.
The exponent does not apply to the first negative sign because the first negative sign lies outside the parentheses.

$$-(-5)^6 = -[(-5) \times (-5) \times (-5) \times (-5) \times (-5) \times (-5)]$$
$$= -(15\ 625)$$
$$= -15\ 625$$

Show You Know

a) Explain how $(-5)^2$ and -5^2 are different and how they are the same.

b) Evaluate $(-6)^2$ and $(-6)^5$.

Key Ideas

- A power is a short way to represent repeated multiplication.
  $7 \times 7 \times 7 = 7^3$
- A power consists of a base and an exponent. The base represents the number you multiply repeatedly. The exponent represents the number of times you multiply the base.

base exponent

$(-3)^5$

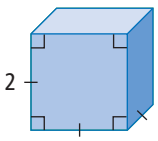
power

Check Your Understanding

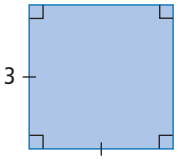
Communicate the Ideas

1. Explain why it is often easier to write an expression as a power rather than as repeated multiplication. Use a specific example.

2. Explain how the two diagrams and calculations show that 2^3 and 3^2 are different.

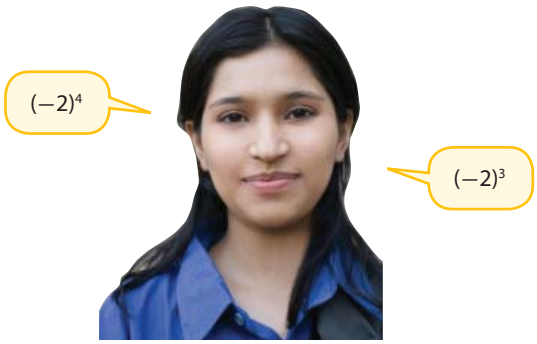


Volume: $2 \times 2 \times 2 = 8$



Area: $3 \times 3 = 9$

3. Pani says, "When you evaluate a power with a negative base and an even exponent, you get a positive value. When you evaluate a power with a negative base and an odd exponent, you get a negative value." Is Pani correct? Justify your answer.



Practise

For help with #4 and #5, refer to Example 1 on page 93.

4. Write each expression as a power, and evaluate.

a) 7×7

b) $3 \times 3 \times 3$

c) $8 \times 8 \times 8 \times 8 \times 8$

d) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

5. Write each expression as a power. Identify the base and the exponent in each power. Then, evaluate.

a) $1 \times 1 \times 1 \times 1$

b) $2 \times 2 \times 2 \times 2 \times 2$

c) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

d) 13

For help with #6 to #9, refer to Example 2 on page 94.

6. Evaluate each power.

a) 5^2 **b)** 3^3 **c)** 4^5

7. What is the value of each power?

a) 8^3 **b)** 2^6 **c)** 1^9

8. Copy and complete the table.

Repeated Multiplication	Exponential Form	Value
a) $6 \times 6 \times 6$	6^3	■
b) $3 \times 3 \times 3 \times 3$	■	■
c) ■	■	49
d) ■	11^2	■
e) ■	■	125

9. Does $4^3 = 3^4$? Show how you know.

For help with #10 to #13, refer to Example 3 on page 95.

10. Evaluate each power.

a) $(-9)^2$ **b)** -5^3 **c)** $(-2)^7$

11. What is the value of each power?

a) -8^2 **b)** $(-1)^5$ **c)** $-(-3)^7$

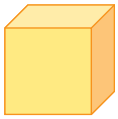
12. Copy and complete the table.

	Repeated Multiplication	Exponential Form	Value
a)	$(-3) \times (-3) \times (-3)$	$(-3)^3$	■
b)	$(-4) \times (-4)$	$(-4)^2$	■
c)	$(-1) \times (-1) \times (-1)$	■	■
d)	■	$(-7)^2$	■
e)	■	■	-1000

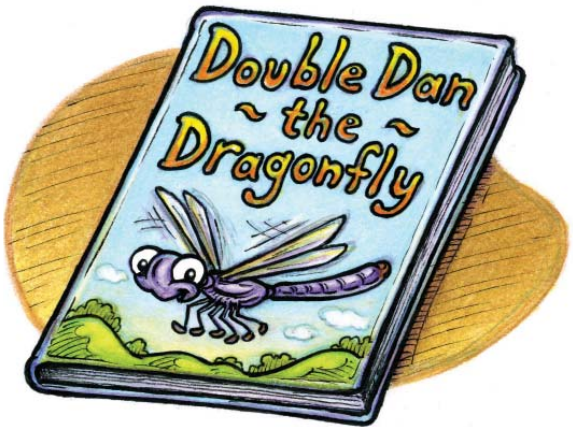
13. Does $(-6)^4 = -6^4$? Show how you know.

Apply

14. The volume of a cube with an edge length of 3 cm is 27 cm^3. Write the volume in repeated multiplication form and exponential form.



15. In a children's story, Double Dan the Dragonfly is growing fast. His body length is doubling every month. At the beginning of the story, his length is 1 cm.



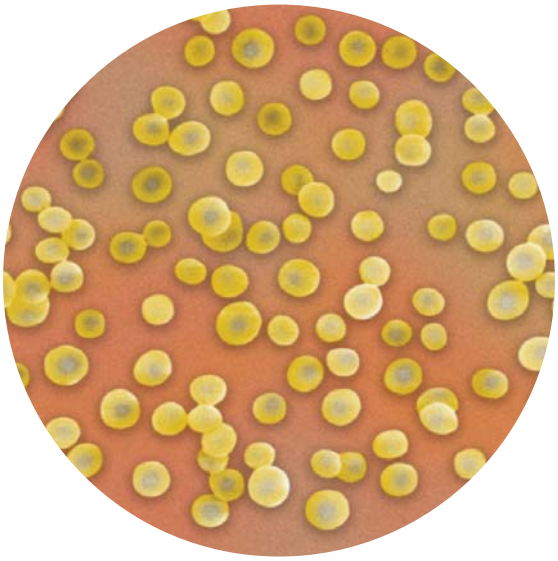
a) Create a table to show how Dan's body length increases every month for ten months.

b) What is his body length five months after the beginning of the story? Express your answer as a power. Then, evaluate.

c) After how many months is his body length more than 50 cm?

16. Arrange the following powers from least to greatest value: 1^{22}, 3^4, 4^3, 2^5, 7^2.

17. A single bacterium doubles in number every hour. How many bacteria are present after 15 h?



18. Express 9 as a power where the exponent is 2 and the base is

a) positive

b) negative

19. Explain what the following statement means using numerical examples:

Multiplication is a way to represent repeated addition, and powers are a way to represent repeated multiplication.

20. The power 7^3 can be read as "seven cubed." Draw a picture of a cube with a volume of 7^3 cubic units, or 343 cubic units. Label appropriate dimensions for the cube.

21. Represent 144 in three different ways using repeated multiplication.

Extend

22. Evaluate the powers of 5 from 5^3 to 5^{10}. Use only whole numbers as exponents.

a) What do you notice about the last three digits of each value?

b) Predict the last three digits if you evaluate 5^{46}.

23. Evaluate the powers of 3 from 3^1 to 3^{12}. Use only whole numbers as exponents.

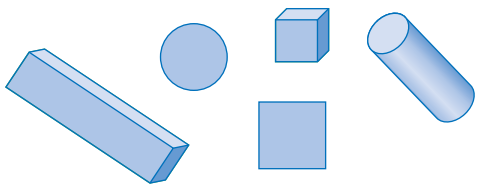
a) What do you notice about the units digit?

b) Predict the units digit if you evaluate 3^{63}. Explain how you arrived at your answer.

Math Link

Some formulas use exponents. Two that you are familiar with are given below.

- $SA = 6s^2$
- $V = \pi r^2 h$



a) Rewrite each formula using repeated multiplication. Identify what the formula represents and how you would use it.

b) For the mobile you will build at the end of the chapter, you will need to use formulas. Identify two formulas that contain exponents, for the shapes shown. Write each formula using repeated multiplication.