2.4 Determining Square Roots of Rational Numbers

Focus on...

After this lesson, you will be able to...

- determine the square root of a perfect square rational number
- determine an approximate square root of a non-perfect square rational number

The Great Pyramid of Giza is the largest of the ancient pyramids in Egypt. The pyramid has a square base with a side length between 230 m and 231 m. Estimate how the dimensions of the base compare with the dimensions of a football field.

Materials

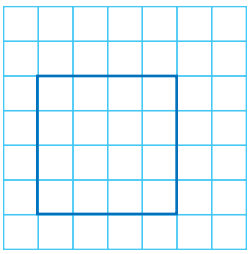
- grid paper

Literacy Link

When the *square root* of a given number is multiplied by itself, the product is the given number. For example, the square root of 9 is 3, because $3 \times 3 = 9$. A square root is represented by the symbol $\sqrt{\ }$, for example, $\sqrt{9} = 3$.

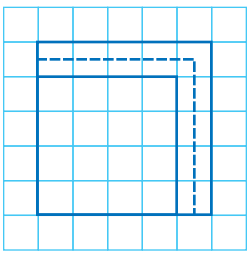
Explore Square Roots of Rational Numbers

1. a) Explain how the diagram represents $\sqrt{16}$.



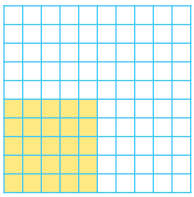
b) Draw a diagram that represents $\sqrt{25}$.

c) Explain how you could use the following diagram to identify a rational number with a square root that is between 4 and 5.



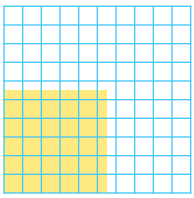
d) Describe another strategy you could use to complete part c).

2. a) Explain how the shading on the hundred grid represents $\sqrt{0.25}$.



b) Draw a diagram that represents $\sqrt{0.36}$.

c) Explain how you could use the following diagram to identify a rational number with a square root that is between 0.5 and 0.6.

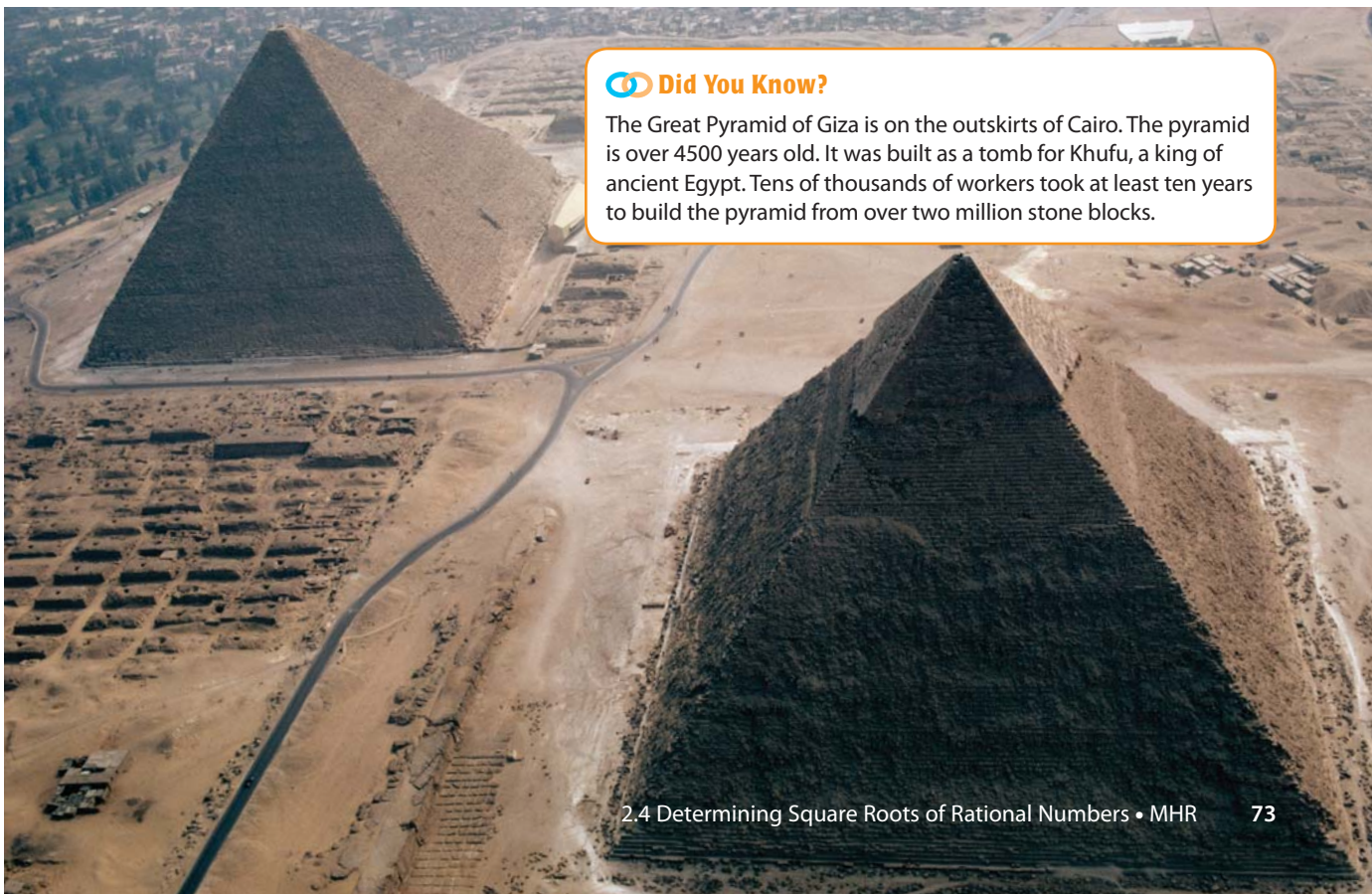


d) Describe another strategy you could use to complete part c).

Reflect and Check

3. Compare your strategies from #1d) and #2d) with a classmate's strategies. How are they similar and different?

4. Use the dimensions provided in the opening paragraph of this section to estimate the base area of the Great Pyramid of Giza. Explain your method.



Did You Know?

The Great Pyramid of Giza is on the outskirts of Cairo. The pyramid is over 4500 years old. It was built as a tomb for Khufu, a king of ancient Egypt. Tens of thousands of workers took at least ten years to build the pyramid from over two million stone blocks.

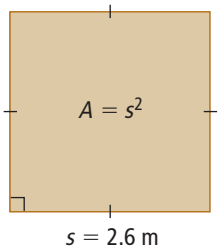
Link the Ideas

Example 1: Determine a Rational Number From Its Square Root

A square trampoline has a side length of 2.6 m. Estimate and calculate the area of the trampoline.

Solution

Strategies
Draw a Diagram



Estimate.
$2^2 = 4$ $\quad 3^2 = 9$
So, 2.6^2 is between 4 and 9.
2.6 is closer to 3 than to 2, so $2.6^2 \approx 7$.
An estimate for the area of the trampoline is 7 m^2.

Calculate.
$2.6^2 = 6.76$ C **2.6** x^2 6.76
The area of a trampoline with a side length of 2.6 m is 6.76 m^2.

Tech Link
Check the key sequence for determining the square of a number on your calculator. If there is no x^2 or equivalent key, just multiply the number by itself.

Show You Know

Estimate and calculate the area of a square photo with a side length of 7.1 cm.

Example 2: Determine Whether a Rational Number Is a Perfect Square

Determine whether each of the following numbers is a perfect square.

a) $\frac{25}{49}$ **b)** 0.4

Solution

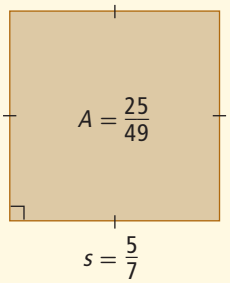
a) In $\frac{25}{49}$, both the numerator and denominator are perfect squares.

How do you know that 25 and 49 are perfect squares?

$\frac{25}{49}$ can be expressed as the product of two equal rational factors, $\frac{5}{7} \times \frac{5}{7}$.

So, $\frac{25}{49}$ is a perfect square.

How does this diagram represent the situation?



b) 0.4 can be expressed in fraction form as $\frac{4}{10}$.

The numerator, 4, is a perfect square.
The denominator, 10, is not a perfect square.
$\frac{4}{10}$ cannot be expressed as the product of two equal rational factors.

How do you know that 10 is not a perfect square?

So, 0.4 is not a perfect square.

Literacy Link

A *perfect square* can be expressed as the product of two equal rational factors. The decimal 0.25 is a perfect square because it can be expressed as 0.5×0.5. The fraction $\frac{9}{16}$ is a perfect square because it can be expressed as $\frac{3}{4} \times \frac{3}{4}$.

Show You Know

Is each of the following numbers a perfect square? Explain.

a) $\frac{121}{64}$ **b)** 1.2 **c)** 0.09

Literacy Link

Square Roots of Perfect Squares

0.25 can be expressed as 0.5×0.5.

Therefore, $\sqrt{0.25} = 0.5$.

$\frac{9}{16}$ can be expressed as $\frac{3}{4} \times \frac{3}{4}$.

Therefore, $\sqrt{\frac{9}{16}} = \frac{3}{4}$.

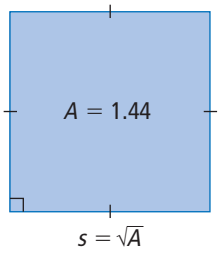
Strategies

Draw a Diagram

Example 3: Determine the Square Root of a Perfect Square

Evaluate $\sqrt{1.44}$.

Solution



Determine the positive number that, when multiplied by itself, results in a product of 1.44.

Method 1: Use Inspection

$1.2 \times 1.2 = 1.44$

So, $\sqrt{1.44} = 1.2$.

Since $12 \times 12 = 144$, then $1.2 \times 1.2 = 1.44$.

Method 2: Use Guess and Check

$1.1 \times 1.1 = 1.21$ Too low

$1.3 \times 1.3 = 1.69$ Too high

$1.2 \times 1.2 = 1.44$ Correct!

So, $\sqrt{1.44} = 1.2$.

Method 3: Use Fraction Form

$$1.44 = \frac{144}{100}$$
$$= \frac{12}{10} \times \frac{12}{10}$$
$$= 1.2 \times 1.2$$

So, $\sqrt{1.44} = 1.2$.

Check: C **1.44** $\sqrt{x}$ 1.2

Tech Link

Check the key sequence for determining square roots on your calculator. Make sure that you can obtain the correct answer for Example 3.

Show You Know

Evaluate.

a) $\sqrt{2.25}$

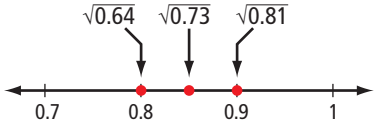
b) $\sqrt{0.16}$

Example 4: Determine a Square Root of a Non-Perfect Square

a) Estimate $\sqrt{0.73}$.
b) Calculate $\sqrt{0.73}$, to the nearest thousandth.

Solution

a) Estimate.
You can use the square root of a perfect square on each side of $\sqrt{0.73}$.
$\sqrt{0.73}$ is about halfway between $\sqrt{0.64}$ and $\sqrt{0.81}$.



One reasonable estimate for $\sqrt{0.73}$ might be about halfway between 0.8 and 0.9, which is about 0.85.

$\sqrt{0.73} \approx 0.85$

b) Calculate.
C 0.73 $\sqrt{x}$ 0.854400375
So, $\sqrt{0.73} \approx 0.854$, to the nearest thousandth.

A calculator usually gives a closer approximation than an estimate does. Why is the value that the calculator shows only an approximation?

Check:
Use the inverse operation, which is squaring.
$0.854^2 = 0.729\ 316$
0.854^2 is close to 0.73.

non-perfect square
- a rational number that cannot be expressed as the product of two equal rational factors
- for example, you cannot multiply any rational number by itself and get an answer of 3, 5, 1.5, or $\frac{7}{8}$

Strategies
Draw a Diagram

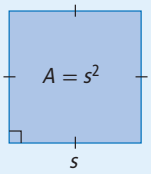
Did You Know?
The square root of a non-perfect square has certain properties:
- It is a non-repeating, non-terminating decimal.
  For example, $\sqrt{5} = 2.236\ 067\ 978\ldots$.
- Its decimal value is approximate, not exact.

Show You Know

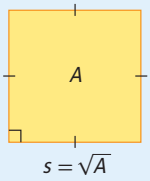
a) Estimate $\sqrt{0.34}$.
b) Calculate $\sqrt{0.34}$, to the nearest thousandth.

Key Ideas

- If the side length of a square models a number, the area of the square models the square of the number.



- If the area of a square models a number, the side length of the square models the square root of the number.



- A perfect square can be expressed as the product of two equal rational factors.
  $3.61 = 1.9 \times 1.9$
  $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$
- The square root of a perfect square can be determined exactly.
  $\sqrt{2.56} = 1.6$
  $\sqrt{\frac{4}{9}} = \frac{2}{3}$
- The square root of a non-perfect square determined using a calculator is an approximation.
  $\sqrt{1.65} \approx 1.284\ 523\ 258$

Check Your Understanding

Communicate the Ideas

1. Max said that the square root of 6.4 is 3.2. Lynda claimed that the square root of 6.4 is 0.8. Jamila estimated that the square root of 6.4 should be about 2.5.

a) Who is correct?

b) What mistakes did the other two students make?

2. Without calculating any square roots, identify the square roots that have values between 4.5 and 5.5. Explain your reasoning.

$\sqrt{21.3}$ $\sqrt{20.1}$ $\sqrt{31.7}$ $\sqrt{27.9}$ $\sqrt{30.5}$ $\sqrt{25.4}$ $\sqrt{30.2}$

3. Since $\sqrt{9}$ is less than 9, and $\sqrt{1.44}$ is less than 1.44, André predicted that $\sqrt{0.0625}$ would be less than 0.0625. Do you agree with his prediction? Explain.

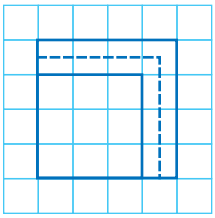
4. a) Determine $\sqrt{2}$ using a scientific calculator. Record all of the digits that the calculator displays.

b) Enter the decimal value of $\sqrt{2}$ from part a) into the calculator and square the value. Record the result.

c) In part a), did the calculator display the exact value of $\sqrt{2}$? Explain how you know.

Practise

5. Use the diagram to identify a rational number with a square root between 3 and 4.



6. Identify a rational number with a square root between 0.7 and 0.8.

For help with #7 and #8, refer to Example 1 on page 74.

7. Estimate and calculate the number that has the given square root.

a) 3.1 **b)** 12.5

c) 0.62 **d)** 0.29

8. Estimate and calculate the area of each square, given its side length.

a) 4.3 cm **b)** 0.035 km

For help with #9 and #10, refer to Example 2 on page 75.

9. Is each of the following rational numbers a perfect square? Explain.

a) $\frac{1}{16}$ **b)** $\frac{5}{9}$

c) 0.36 **d)** 0.9

10. Determine whether each rational number is a perfect square.

a) $\frac{7}{12}$ **b)** $\frac{100}{49}$

c) 0.1 **d)** 0.01

For help with #11 and #12, refer to Example 3 on page 76.

11. Evaluate.

a) $\sqrt{324}$ **b)** $\sqrt{2.89}$

c) $\sqrt{0.0225}$ **d)** $\sqrt{2025}$

12. Calculate the side length of each square from its area.

a) 169 m^2 **b)** 0.16 mm^2

For help with #13 and #14, refer to Example 4 on page 77.

13. Estimate each square root. Then, calculate it to the specified number of decimal places.

a) $\sqrt{39}$, to the nearest tenth

b) $\sqrt{4.5}$, to the nearest hundredth

c) $\sqrt{0.87}$, to the nearest thousandth

d) $\sqrt{0.022}$, to the nearest thousandth

14. Given the area of each square, determine its side length. Express your answer to the nearest hundredth of a unit.

a) 0.85 m^2 **b)** 60 cm^2

Apply

15. Kai needs to replace the strip of laminate that is glued to the vertical faces on a square tabletop. The tabletop has an area of 1.69 m^2. What length of laminate does she need?

16. **a)** The label on a 1-L can of paint states that the paint will cover an area of 10 m^2. What is the side length of the largest square area that the paint will cover? Express your answer to the nearest hundredth of a metre.

b) What is the side length of the largest square area that a 3.79-L can of the same paint will cover? Express your answer to the nearest hundredth of a metre.

c) Nadia is applying two coats of the paint to an area that is 4.6 m by 4.6 m. How much paint will she use if she applies the same amount of paint for each coat? Express your answer to the nearest tenth of a litre.

17. Some parks contain fenced gardens. Suppose that it costs $80 to build each metre of fence, including materials and labour.

a) How much does it cost to enclose a square with an area of 120 m^2? Express your answer to the nearest dollar.

b) Predict whether the total cost of enclosing two squares with an area of 60 m^2 each is the same as your answer to part a).

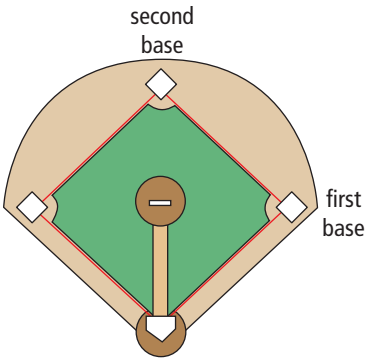
c) Test your prediction from part b) and describe your findings.

18. A frame measures 30 cm by 20 cm. Can you mount a square picture with an area of 500 cm^2 in the frame? Explain.

19. A square picture with an area of 100 cm^2 is mounted on a square piece of matting. The matting has 2.5 times the area of the picture. If the picture is centred on the matting, what width of matting is visible around the outside of the picture? Give your answer to the nearest tenth of a centimetre.

20. Leon's rectangular living room is 8.2 m by 4.5 m. A square rug covers $\frac{2}{5}$ of the area of the floor. What is the side length of the rug, to the nearest tenth of a metre?

21. A baseball diamond is a square area of about 750 m^2. What is the distance from first to second base. Give your answer to the nearest tenth of a metre.



22. The hypotenuse of an isosceles right triangle has a length of 20 cm. What is the length of each leg of the triangle? Provide your answer to the nearest tenth of a centimetre.

23. A rectangular floor that measures 3 m by 2 m is covered by 384 square tiles. Determine the side length of each tile, in centimetres. State any assumptions you make.

24. The distance, d, in kilometres, that a person can see across the ocean to the horizon is given by the formula $d = \sqrt{12.74 \times h}$. In the formula h is the height, in metres, of the person's eyes above the water. Calculate the distance that each of the following people can see across the ocean to the horizon. Express each answer to the nearest tenth of a kilometre.

a) Adèle is sitting on a lifeguard station at the edge of the ocean. Her eyes are 4.1 m above the water.

b) Brian is standing at the water's edge. His eyes are 165 cm above the water.

c) Yvonne is the pilot of an aircraft flying 5 km above the coastline.

Literacy Link

Perform operations under a square root symbol before taking the square root. For example, $\sqrt{9 \times 4} = \sqrt{36}$ or 6.

25. What is the length of the longest line segment you can draw on a sheet of paper that is 27.9 cm by 21.6 cm? Express your answer to the nearest tenth of a centimetre.

26. A bag of fertilizer will cover an area of 200 m^2. Determine the dimensions of a square that $\frac{3}{4}$ of a bag of fertilizer will cover. Express your answer to the nearest tenth of a metre.

27. The surface area of a cube is 100 cm^2. Determine the edge length of the cube, to the nearest tenth of a centimetre.

28. The period, t, in seconds, of a pendulum is the time it takes for a complete swing back and forth. The period can be calculated from the length, l, in metres, of the pendulum using the formula $t = \sqrt{4l}$. Determine the period of a pendulum with each of the following lengths. Express each answer to the nearest hundredth of a second.

a) 1.6 m **b)** 2.5 m **c)** 50 cm

29. The speed of sound in air, s, in metres per second, is related to the temperature, t, in degrees Celsius, by the formula $s = \sqrt{401(273 + t)}$. How much greater is the speed of sound on a day when the temperature is 30 °C than on a day when the temperature is −20 °C? Express your answer to the nearest metre per second.

30. A square field has an area of 1000 m^2. Laura wants to walk from one corner of the field to the opposite corner. If she walks at 1.5 m/s, how much time can she save by walking diagonally instead of walking along two adjacent sides? Express your answer to the nearest tenth of a second.

31. The area of a triangle can be determined using Heron's formula, which requires the side lengths of the triangle. Heron's formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$. In the formula A is the area; a, b, and c are the side lengths; and s is half the perimeter or $\frac{a+b+c}{2}$. Determine the area of each triangle with the following side lengths. Express each area to the nearest tenth of a square centimetre.

a) 15 cm, 12 cm, 10 cm

b) 9.3 cm, 11.4 cm, 7.5 cm

History Link

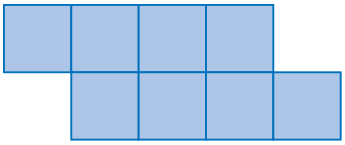
Heron's formula was determined by a Greek mathematician and inventor named Heron (or Hero) of Alexandria. Historians believe that he lived in the first century of the Common Era, but little is known of his life. Heron wrote about geometry and about his inventions, which included machines driven by water, steam, and compressed air.

WWW Web Link

For more information about Heron of Alexandria, go to www.mathlinks9.ca and follow the links.

Extend

32. This shape is made from eight congruent squares. The total area of the shape is 52 cm^2. What is its perimeter, to the nearest centimetre?



33. A square has an area of 32 cm^2. What is the area of the largest circle that will fit inside it? Express your answer to the nearest tenth of a square centimetre.

34. Use the formula $r = \sqrt{\frac{A}{\pi}}$ to determine the radius of a circular garden with an area of 40 m^2. Express your answer to the nearest tenth of a metre.

35. The width of a rectangle is $\frac{1}{3}$ its length. The area of the rectangle is 9.72 cm^2. What are the dimensions of the rectangle?

36. Determine $\sqrt{\sqrt{\sqrt{65\,536}}}$.

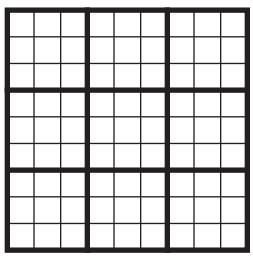
Math Link

Sudoku is a Japanese logic puzzle completed on a 9-by-9 square grid. The grid includes nine 3-by-3 sections.

Answer each of the following questions about the sudoku grid in two different ways. Compare your solutions with your classmates' solutions.

a) If the smallest squares on the grid have a side length of 1.1 cm, what is the area of the whole grid?

b) If the whole grid has an area of 182.25 cm^2, what are the dimensions of each 3 by 3 section?



WWW Web Link

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