2.3 Problem Solving With Rational Numbers in Fraction Form

Focus on...

After this lesson, you will be able to...

- perform operations on rational numbers in fraction form
- solve problems involving rational numbers in fraction form

Literacy Link

Klassen's winning time of 1:55.27 means 1 min, 55.27 s.

Web Link

To learn more about Cindy Klassen and other Canadian speed skaters, go to www.mathlinks9.ca and follow the links.

A news report gives the results of an Olympic speed skating event:

> Winnipeg's Cindy Klassen won the gold medal in the 1500-m speed skating event at the Winter Olympics in Turin, Italy. Her winning time was 1:55.27. Ottawa's Kristina Groves won the silver medal. She finished in a time of 1:56.74. The bronze medalist was Ireen Wust of the Netherlands. She finished only $\frac{16}{100}$ s behind Groves.

What are Klassen's time and Groves's time as a mixed number of seconds? How many seconds, in decimal form, did Wust finish behind Groves?

Explore Adding and Subtracting Rational Numbers

1. Determine the number of seconds by which Klassen beat Groves
   - **a)** by using their times in fraction form to give an answer in fraction form
   - **b)** by using their times in decimal form to give an answer in decimal form
2. Which method did you prefer in #1? Explain.
3. What was Wust's time for the event? Show your work.
4. By how many seconds did Klassen beat Wust? Use fractions to show two ways to determine your answer.

Reflect and Check

5. **a)** Use the following data to create a problem involving the addition or subtraction of rational numbers. Ask a partner to solve it.

> Canada's Perdita Felicien won in the 100-m hurdles at the world championships in Paris, France. Her time was 12.53 s. Brigitte Foster-Hylton of Jamaica placed second at 12.57 s. Foster-Hylton was $\frac{1}{10}$ s ahead of Miesha McKelvy of the United States.

   b) Show how you could determine the answer to your problem by using a different method than your partner used.

   c) Discuss with your partner the differences between your methods. Decide which method you prefer. Explain your choice.

Link the Ideas

Example 1: Add and Subtract Rational Numbers in Fraction Form

Estimate and calculate.

a) $\frac{2}{5} - \left(-\frac{1}{10}\right)$

b) $3\frac{2}{3} + \left(-1\frac{3}{4}\right)$

Solution

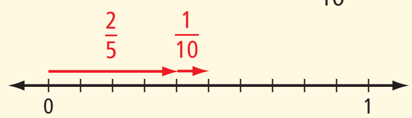
a) Estimate. $\frac{1}{2} - 0 = \frac{1}{2}$

ME

Calculate.

$$\frac{2}{5} - \left(-\frac{1}{10}\right)$$
$$= \frac{2}{5} - \left(\frac{-1}{10}\right)$$
$$= \frac{4}{10} - \left(\frac{-1}{10}\right)$$
$$= \frac{4 - (-1)}{10}$$
$$= \frac{5}{10}$$
$$= \frac{1}{2}$$

Subtracting $-\frac{1}{10}$ is the same as adding the opposite of $-\frac{1}{10}$.



A common denominator of 5 and 10 is 10.

Is the calculated answer close to the estimate?

b) Estimate. $4 + (-2) = 2$

ME

Calculate.

Method 1: Rewrite the Mixed Numbers as Improper Fractions

$$3\frac{2}{3} + \left(-1\frac{3}{4}\right) = \frac{11}{3} + \left(-\frac{7}{4}\right)$$

Add.

$$\frac{11}{3} + \left(-\frac{7}{4}\right)$$
$$= \frac{11}{3} + \left(\frac{-7}{4}\right)$$
$$= \frac{44}{12} + \left(\frac{-21}{12}\right)$$
$$= \frac{44 + (-21)}{12}$$
$$= \frac{23}{12}$$
$$= 1\frac{11}{12}$$

Method 2: Add the Integers and Add the Fractions

$$3\frac{2}{3} + \left(-1\frac{3}{4}\right)$$
$$= 3 + \frac{2}{3} + (-1) + \left(-\frac{3}{4}\right)$$
$$= 3 + (-1) + \frac{2}{3} + \left(-\frac{3}{4}\right)$$
$$= 3 + (-1) + \frac{8}{12} + \left(-\frac{9}{12}\right)$$
$$= 2 + \left(-\frac{1}{12}\right)$$
$$= 1\frac{12}{12} + \left(-\frac{1}{12}\right)$$
$$= 1\frac{11}{12}$$

Show You Know

Estimate and calculate.

a) $-\frac{3}{4} - \frac{1}{5}$

b) $-2\frac{1}{2} + 1\frac{9}{10}$

Example 2: Multiply and Divide Rational Numbers in Fraction Form

Determine.

a) $\frac{3}{4} \times \left(-\frac{2}{3}\right)$

b) $-1\frac{1}{2} \div \left(-2\frac{3}{4}\right)$

Solution

a) Multiply the numerators and multiply the denominators.

$$\frac{3}{4} \times \left(-\frac{2}{3}\right) = \frac{3}{4} \times \left(\frac{-2}{3}\right) = \frac{3 \times (-2)}{4 \times 3} = \frac{-6}{12} = \frac{-1}{2} \text{ or } -\frac{1}{2}$$

$1 \times \left(-\frac{1}{2}\right) = -\frac{1}{2}$ ME

You could remove the common factors of 3 and 2 from the numerator and denominator before multiplying.

$$\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \left(\frac{\cancel{-2}^{-1}}{\cancel{3}_{1}}\right) = \frac{-1}{2} \text{ or } -\frac{1}{2}$$

b) *Method 1: Use a Common Denominator*

Write the fractions with a common denominator and divide the numerators.

$$-1\frac{1}{2} \div \left(-2\frac{3}{4}\right) = -\frac{3}{2} \div \left(-\frac{11}{4}\right) = \frac{-6}{4} \div \left(\frac{-11}{4}\right) = \frac{-6}{-11} = \frac{6}{11}$$

Recall that $\frac{-6}{4} \div \left(\frac{-11}{4}\right) = \frac{-6 \div (-11)}{4 \div 4} = \frac{-6 \div (-11)}{1} = \frac{-6}{-11}$

Method 2: Multiply by the Reciprocal

Another strategy is to multiply by the reciprocal.

$$\frac{-3}{2} \div \left(\frac{-11}{4}\right) = \frac{-3}{2} \times \frac{4}{-11} = \frac{-12}{-22} = \frac{6}{11}$$

You could remove the common factor of 2 from the numerator and denominator.

$$\frac{-3}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{-11} = \frac{6}{11}$$

Show You Know

Determine each value.

a) $-\frac{2}{5}\left(-\frac{1}{6}\right)$

b) $-2\frac{1}{8} \div 1\frac{1}{4}$

Example 3: Apply Operations With Rational Numbers in Fraction Form

At the start of a week, Maka had $30 of her monthly allowance left. That week, she spent $\frac{1}{5}$ of the money on bus fares, another $\frac{1}{2}$ shopping, and $\frac{1}{4}$ on snacks. How much did she have left at the end of the week?

Solution

You can represent the $30 Maka had at the beginning of the week by 30.

You can represent the fractions of the money spent by $-\frac{1}{5}$, $-\frac{1}{2}$, and $-\frac{1}{4}$.

> Why would you represent the $30 by a positive rational number?
>
> Why would you represent the fractions of money spent by negative rational numbers?

Calculate each dollar amount spent.

For bus fares:

$$-\frac{1}{5} \times 30$$
$$= \frac{-1}{5} \times \frac{30}{1}$$
$$= \frac{-30}{5}$$
$$= -6$$

For shopping:

$$-\frac{1}{2} \times 30$$
$$= \frac{-1}{2} \times \frac{30}{1}$$
$$= \frac{-30}{2}$$
$$= -15$$

For snacks:

$$-\frac{1}{4} \times 30$$
$$= \frac{-1}{4} \times \frac{30}{1}$$
$$= \frac{-30}{4}$$
$$= -\frac{15}{2} \text{ or } -7.5$$

Determine the total dollar amount spent.
$-6 + (-15) + (-7.5) = -28.5$

Determine how much Maka had left.
$30 + (-28.5) = 1.5$

Maka had $1.50 left at the end of the week.

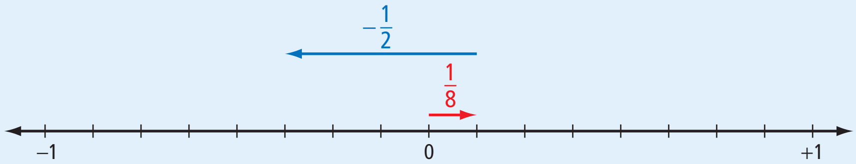
> You could also calculate the total by adding the three fractions $-\frac{1}{5}$, $-\frac{1}{2}$, and $-\frac{1}{4}$ and multiplying their sum by 30. Why does this strategy work? Which strategy do you prefer?

Show You Know

Stefano had $46 in a bank account that he was not using. Each month for three months, the bank withdrew $\frac{1}{4}$ of this amount as a service fee. How much was left in the account after the last withdrawal?

Key Ideas

- The addition of rational numbers can be modelled on a number line. The subtraction of rational numbers can be modelled by adding the opposite on a number line.



$\frac{1}{8} + \left(-\frac{1}{2}\right) = -\frac{3}{8}$ or $\frac{1}{8} - \frac{1}{2} = -\frac{3}{8}$

- Rational numbers expressed as proper or improper fractions can be added and subtracted in the same way as positive fractions.
- Rational numbers expressed as mixed numbers can be added by
  - first writing them as improper fractions
  - adding the integers and adding the fractions
- Rational numbers expressed as mixed numbers can be subtracted by first writing them as improper fractions.
- Rational numbers expressed as proper or improper fractions can be multiplied and divided in the same way as positive fractions. The sign of the product or quotient can be predicted from the sign rules for multiplication and division.
- Rational numbers expressed as mixed numbers can be multiplied and divided by first writing them as improper fractions.

Check Your Understanding

Communicate the Ideas

1. Emma and Oleg both calculated $-\frac{1}{12} - \frac{2}{3}$ correctly.

a) Emma used 12 as a common denominator. Show how she calculated the difference in lowest terms.

b) Oleg used 36 as a common denominator. Show how he calculated the difference in lowest terms.

c) Which common denominator do you prefer using for the calculation? Explain.

2. Ming and Al both determined $-\frac{7}{15} \times \left(-\frac{5}{14}\right)$ and wrote the product in lowest terms. Ming multiplied before she removed common factors. Al removed common factors before he multiplied. Whose method do you prefer? Explain.

3. a) Calculate $\frac{-9}{12} \div \frac{3}{8}$ by multiplying by the reciprocal.

b) Calculate $\frac{-9}{12} \div \frac{3}{8}$ by writing the fractions with a common denominator and dividing the numerators.

c) Which method do you prefer for this calculation? Explain.

4. Joshua suggested his own method for multiplying and dividing rational numbers in fraction form. For $-\frac{4}{5} \times \frac{2}{3}$, he calculated $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$. Then, he reasoned that the product must be negative because $-\frac{4}{5}$ and $\frac{2}{3}$ have different signs. He gave the answer as $-\frac{8}{15}$. Describe an advantage and a disadvantage of Joshua's method.

Practise

For help with #5 and #6, refer to Example 1 on page 64. Write your answers in lowest terms.

5. Estimate and calculate.

a) $\frac{3}{10} + \frac{1}{5}$

b) $2\frac{1}{3} + \left(-1\frac{1}{4}\right)$

c) $-\frac{5}{12} - \frac{5}{12}$

d) $-2\frac{1}{2} - \left(-3\frac{1}{3}\right)$

e) $-\frac{5}{6} + \frac{1}{3}$

f) $\frac{3}{8} - \left(-\frac{1}{4}\right)$

6. Estimate and calculate.

a) $\frac{2}{3} - \frac{3}{4}$

b) $-\frac{2}{9} + \left(-\frac{1}{3}\right)$

c) $-\frac{1}{4} + \left(-\frac{3}{5}\right)$

d) $-\frac{3}{4} - \left(-\frac{5}{8}\right)$

e) $1\frac{1}{2} - 2\frac{1}{4}$

f) $1\frac{2}{5} + \left(-1\frac{3}{4}\right)$

For help with #7 and #8, refer to Example 2 on page 65. Write your answers in lowest terms.

7. Estimate and calculate.

a) $\frac{4}{5} \div \frac{5}{6}$

b) $3\frac{1}{3}\left(1\frac{3}{4}\right)$

c) $\frac{1}{8} \times \left(-\frac{2}{5}\right)$

d) $-\frac{9}{10} \div \left(-\frac{4}{5}\right)$

e) $-\frac{3}{8} \times 5\frac{1}{3}$

f) $\frac{1}{10} \div \left(-\frac{3}{8}\right)$

8. Estimate and calculate.

a) $-\frac{3}{4} \times \left(-\frac{1}{9}\right)$

b) $1\frac{1}{3} \div 1\frac{1}{4}$

c) $-\frac{3}{8} \div \frac{7}{10}$

d) $-2\frac{1}{8} \div 1\frac{1}{4}$

e) $\frac{7}{9}\left(-\frac{6}{11}\right)$

f) $-1\frac{1}{2} \div \left(-2\frac{1}{2}\right)$

For help with #9 and #10, refer to Example 3 on page 66.

9. Lori owed her mother \$39. Lori paid back $\frac{1}{3}$ of this debt and then paid back $\frac{1}{4}$ of the remaining debt.
How much does Lori still owe her mother?

10. A carpenter has 64 m of baseboard. He installs $\frac{1}{2}$ of the baseboard in one room. He installs another $\frac{3}{5}$ of the original amount of baseboard in another room. How much baseboard does he have left?

Apply

11. In everyday speech, *in a jiffy* means in a very short time. In science, a specific value sometimes assigned to a jiffy is $\frac{1}{100}$ s.
Naima can type at 50 words/min. On average, how many jiffies does she take to type each word?

12. In the table, a positive number shows how many hours the time in a location is ahead of the time in London, England. A negative number shows how many hours the time is behind the time in London.

Location	Time Zone
Alice Springs, Australia	$+9\frac{1}{2}$
Brandon, Manitoba	-6
Chatham Islands, New Zealand	$+12\frac{3}{4}$
Istanbul, Turkey	$+2$
Kathmandu, Nepal	$+5\frac{3}{4}$
London, England	0
Mumbai, India	$+5\frac{1}{2}$
St. John's, Newfoundland and Labrador	$-3\frac{1}{2}$
Tokyo, Japan	$+9$
Victoria, British Columbia	-8

a) How many hours is the time in St. John's ahead of the time in Brandon?

b) How many hours is the time in Victoria behind the time in Mumbai?

c) Determine and interpret the time difference between Tokyo and Kathmandu.

d) Determine and interpret the time difference between Chatham Islands and St. John's.

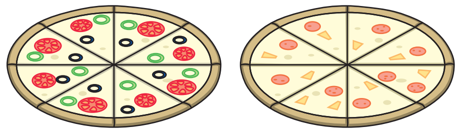
e) In which location is the time exactly halfway between the times in Istanbul and Alice Springs?

13. The diameter of Pluto is $\frac{6}{17}$ the diameter of Mars. Mars is $\frac{17}{300}$ the diameter of Saturn.

a) What fraction of the diameter of Saturn is the diameter of Pluto?

b) The diameter of Saturn is 120 000 km. What is the diameter of Pluto?

14. Li and Ray shared a vegetarian pizza and a Hawaiian pizza of the same size. The vegetarian pizza was cut into eight equal slices. The Hawaiian pizza was cut into six equal slices. Li ate two slices of the vegetarian pizza and one slice of the Hawaiian pizza. Ray ate two slices of the Hawaiian pizza and one slice of the vegetarian pizza.



a) Who ate more pizza?

b) How much more did that person eat?

c) How much pizza was left over?

15. Predict the next three numbers in each pattern.

a) $-1\frac{1}{2}, -\frac{7}{8}, -\frac{1}{4}, \frac{3}{8}, 1, \ldots$

b) $1\frac{1}{3}, -\frac{2}{3}, \frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, \ldots$

16. Boris has $2\frac{1}{2}$ times as much cash as Anna. Charlie has $\frac{3}{4}$ as much cash as Anna. Anna has $25.60 in cash.

a) How much cash do the three people have altogether?

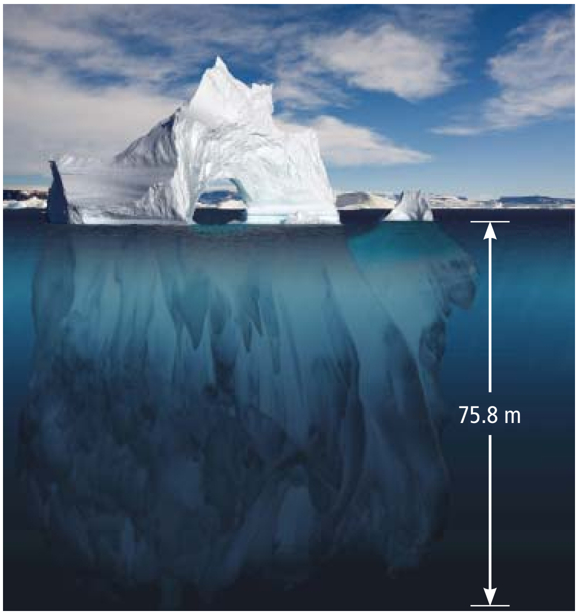
b) How much more cash does Boris have than Charlie?

17. To calculate $-\frac{3}{4} + \left(-\frac{2}{3}\right)$, Amy decided to convert the fractions to decimals and add the decimals on a scientific calculator.

a) Explain why she had difficulty in determining the exact answer by this method.

b) How should she calculate to get an exact answer?

18. One ninth of the height of an iceberg was above the surface of the water. The iceberg extended to a depth of 75.8 m below the surface. What was the height of the iceberg above the surface? Express your answer to the nearest tenth of a metre.



19. Copy and complete each statement.

a) $\frac{1}{2} + ■ = -\frac{3}{4}$ **b)** $■ - 1\frac{4}{5} = -\frac{7}{10}$

c) $-2\frac{1}{6} \times ■ = -1\frac{1}{3}$ **d)** $■ \div \left(-\frac{3}{5}\right) = 2\frac{1}{2}$

20. In a magic square, the sum of each row, column, and diagonal is the same. Copy and complete this magic square.

$-\frac{1}{2}$		
	$-\frac{5}{6}$	
	$\frac{1}{2}$	$-1\frac{1}{6}$

21. Calculate.

a) $\frac{1}{3}\left(\frac{2}{5} - \frac{1}{2}\right) + \frac{3}{10}$

b) $\frac{3}{4} \div \frac{5}{8} - \frac{3}{8} \div \frac{1}{2}$

c) $1\frac{1}{2} + 1\frac{1}{2}\left(-2\frac{5}{6} + \frac{1}{3}\right)$

22. Taj has three scoops for measuring flour. The largest scoop holds $2\frac{1}{2}$ times as much as the smallest one. The middle scoop holds $1\frac{3}{4}$ times as much as the smallest one. Describe two different ways in which Taj could measure each of the following quantities. He can use full scoops only.

a) $3\frac{1}{4}$ times as much as the smallest scoop holds

b) $\frac{1}{2}$ as much as the smallest scoop holds

23. **a)** Write a subtraction statement involving two negative fractions or negative mixed numbers so that the difference is $-\frac{4}{3}$.

b) Share your statement with a classmate.

Extend

24. Can the sum of two rational numbers be less than both of the rational numbers? Explain using examples in fraction form.

25. The following expression has a value of 1.

$$\left[-\frac{1}{2} + \left(-\frac{1}{2}\right)\right] \div \left[-\frac{1}{2} + \left(-\frac{1}{2}\right)\right]$$

Use $-\frac{1}{2}$ four times to write expressions with each of the following values.

a) -1 **b)** 0

c) $\frac{1}{4}$ **d)** 4

e) $-\frac{3}{4}$ **f)** $-1\frac{1}{2}$

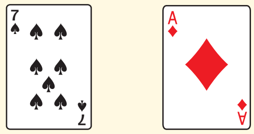
26. Multiplying a fraction by $-\frac{1}{2}$, then adding $\frac{3}{4}$, and then dividing by $-\frac{1}{4}$ gave an answer of $-3\frac{3}{4}$. What was the original fraction?

27. For what values of x does $x - \frac{1}{x} = 1\frac{1}{2}$?

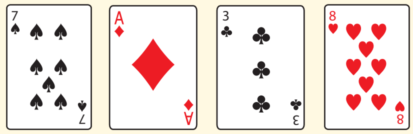
Math Link

Play this game with a partner or in a small group. You will need a deck of playing cards.

- Remove the jokers, face cards, and 10s from the deck.
- Red cards represent positive integers. Black cards represent negative integers. Aces represent 1 or −1.
- In each round, the dealer shuffles the cards and deals four cards to each player.
- Use your four cards to make two fractions with a product that is as far from zero as possible.
- In each round, the player with the product that is furthest from zero wins two points. If there is a tie, each tied player wins a point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.



represents −7 represents 1



represent −7, 1, −3, and 8

An expression for the product furthest from zero is

$\frac{-7}{1} \times \frac{8}{-3}$ or $\frac{8}{-3} \times \frac{-7}{1}$ or $\frac{8}{1} \times \frac{-7}{-3}$ or $\frac{-7}{-3} \times \frac{8}{1}$

History Link

Fractions in Ancient Egypt

A fraction with a numerator of 1, such as $\frac{1}{4}$, is called a *unit fraction*. In ancient Egypt, fractions that were not unit fractions were expressed as sums of unit fractions. For example, $\frac{5}{6}$ was expressed as $\frac{1}{2} + \frac{1}{3}$.

1. Express each of the following as the sum of two unit fractions.

a) $\frac{3}{10}$ **b)** $\frac{9}{14}$ **c)** $\frac{9}{20}$ **d)** $\frac{11}{18}$

2. Describe any strategies that helped you to complete #1.

Did You Know?

The Eye of Horus as shown below was used by ancient Egyptians to represent fractions. Each part of the Eye of Horus was a unit fraction. Egyptians believed that the parts had a combined value of 1.

Repetition was not allowed in the Egyptian system. Therefore, $\frac{2}{5}$ was not expressed as $\frac{1}{5} + \frac{1}{5}$ but could be expressed as $\frac{1}{15} + \frac{1}{3}$.

3. Express each of the following as the sum of two unit fractions without using any fraction more than once.

a) $\frac{2}{7}$ **b)** $\frac{2}{9}$ **c)** $\frac{2}{11}$

4. Express each of the following as the sum of three unit fractions without using any fraction more than once.

a) $\frac{7}{8}$ **b)** $\frac{11}{24}$ **c)** $\frac{3}{4}$