

Problem Solving With Rational Numbers in Decimal Form

Focus on...

After this lesson, you will be able to...

- perform operations on rational numbers in decimal form
- solve problems involving rational numbers in decimal form



Did You Know?

As Canada's sunniest provincial capital, Regina averages almost 6.5 h of sunshine per day. That is over 2 h per day more sunshine than St. John's, Newfoundland and Labrador. St. John's is the least sunny provincial capital.

In Regina, Saskatchewan, the average mid-afternoon temperature in January is -12.6°C . The average mid-afternoon temperature in July is 26.1°C . Estimate how much colder Regina is in January than in July.

Explore Multiplying and Dividing Rational Numbers in Decimal Form

- Estimate the products and quotients. Explain your method.
 3.2×4.5 $3.2 \times (-4.5)$ $-20.9 \div 9.5$ $-20.9 \div (-9.5)$
 - Calculate the products and quotients in part a). Explain your method.
- Suppose the temperature one January afternoon in Regina decreased by 2.6°C every hour for 3.5 h. What was the overall temperature change during that time?
 - Suppose the temperature in Regina one July afternoon increased by 9.9°C in 5.5 h. What was the average temperature change per hour?

Reflect and Check

- How can you multiply and divide rational numbers in decimal form?
- Create a problem that can be solved using the multiplication or division of rational numbers. Challenge a classmate to solve it.

Link the Ideas

Example 1: Add and Subtract Rational Numbers in Decimal Form

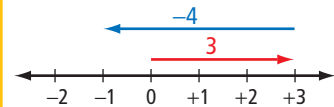
Estimate and calculate.

- a) $2.65 + (-3.81)$
 b) $-5.96 - (-6.83)$

Solution

- a) Estimate.
 $2.65 + (-3.81)$
 $\approx 3 + (-4)$
 ≈ -1

How does the number line show the estimate?



Method 1: Use Paper and Pencil

Calculate.

Adding the opposite of 3.81 is the same as subtracting 3.81.

$$2.65 + (-3.81) = 2.65 - 3.81$$

Determine the difference between 3.81 and 2.65.

$$3.81 - 2.65 = 1.16$$

However, $2.65 - 3.81$ must be negative, since $3.81 > 2.65$.

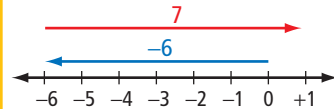
$$\text{So, } 2.65 + (-3.81) = -1.16.$$

Method 2: Use a Calculator

$$\text{Calculate. } \boxed{C} \boxed{2.65} \boxed{+} \boxed{3.81} \boxed{+/-} \boxed{=} \boxed{-1.16}$$

- b) Estimate.
 $-5.96 - (-6.83)$
 $\approx -6 - (-7)$
 $\approx -6 + 7$
 ≈ 1

How does the number line show the estimate?



Method 1: Use Paper and Pencil

Calculate.

$$-5.96 - (-6.83) = -5.96 + 6.83$$

Determine $6.83 + (-5.96)$.

$$\begin{aligned} 6.83 + (-5.96) \\ = 6.83 - 5.96 \\ = 0.87 \end{aligned}$$

$$\text{So, } -5.96 - (-6.83) = 0.87.$$

Why is subtracting -6.83 the same as adding its opposite, 6.83 ?

Why is $-5.96 + 6.83$ the same as $6.83 + (-5.96)$?

Is the calculated answer reasonable? How do you know?

Method 2: Use a Calculator

$$\text{Calculate. } \boxed{C} \boxed{5.96} \boxed{+/-} \boxed{-} \boxed{6.83} \boxed{+/-} \boxed{=} \boxed{0.87}$$

Tech Link

The key for entering a negative sign may look different on different calculators, for example, $\boxed{+/-}$, $\boxed{+/-}$ or $\boxed{(-)}$. It is not the subtraction key, $\boxed{-}$. Experiment with calculations involving negative signs on your calculator.

Show You Know

Estimate and calculate.

- a) $-4.38 + 1.52$
- b) $-1.25 - 3.55$

Example 2: Multiply and Divide Rational Numbers in Decimal Form

Estimate and calculate.

- a) $0.45 \times (-1.2)$
- b) $-2.3 \div (-0.25)$

Solution

- a) Estimate.

$$0.5 \times (-1) = -0.5$$



Calculate.

Method 1: Use Paper and Pencil

You can calculate by multiplying the decimal numbers.

$$0.45 \times 1.2 = 0.54$$

Determine the sign of the product.

$$0.45 \times (-1.2) = -0.54$$

How do you know what the sign of the product is?

Method 2: Use a Calculator

$$\text{C } 0.45 \times 1.2 + \text{C} = -0.54$$

- b) Estimate.

$$\begin{aligned} -2.3 \div (-0.25) & \approx -2 \div (-0.2) \\ & \approx 10 \end{aligned}$$



Calculate.

$$\text{C } 2.3 + \text{C} \div 0.25 + \text{C} = 9.2$$

Show You Know

Estimate and calculate.

- a) $-1.4(-2.6)$
- b) $-2.76 \div 4.6$

Literacy Link

Parentheses is another name for brackets. They can be used in place of a multiplication sign.

For example,

$$-4 \times 1.5 = -4(1.5)$$

Example 3: Apply Operations With Rational Numbers in Decimal Form

On Saturday, the temperature at the Blood Reserve near Stand Off, Alberta decreased by $1.2\text{ }^{\circ}\text{C/h}$ for 3.5 h. It then decreased by $0.9\text{ }^{\circ}\text{C/h}$ for 1.5 h.



- What was the total decrease in temperature?
- What was the average rate of decrease in temperature?

Solution

- The time periods can be represented by 3.5 and 1.5. The rates of temperature decrease can be represented by -1.2 and -0.9 .

Method 1: Calculate in Stages

You can represent the temperature decrease in the first 3.5 h by $3.5 \times (-1.2) = -4.2$.

You can represent the temperature decrease in the last 1.5 h by $1.5 \times (-0.9) = -1.35$.

Add to determine the total temperature decrease.
 $-4.2 + (-1.35) = -5.55$

The total decrease in temperature was $5.55\text{ }^{\circ}\text{C}$.

Method 2: Evaluate One Expression

The total temperature decrease can be represented by
 $3.5 \times (-1.2) + 1.5 \times (-0.9)$.

Evaluate this expression, using the order of operations.

$$\begin{aligned} & 3.5 \times (-1.2) + 1.5 \times (-0.9) \\ &= -4.2 + (-1.35) \\ &= -5.55 \end{aligned}$$

You can also use a calculator.

$$\boxed{C} \boxed{3.5} \boxed{\times} \boxed{1.2} \boxed{+/-} \boxed{+} \boxed{1.5} \boxed{\times} \boxed{0.9} \boxed{+/-} \boxed{=} \boxed{-5.55}$$

The total decrease in temperature was $5.55\text{ }^{\circ}\text{C}$.

- The average rate of decrease in temperature is the total decrease divided by the total number of hours.

The total number of hours is $3.5 + 1.5 = 5$.

$$\frac{-5.55}{5} = -1.11$$

The average rate of decrease in temperature was $1.11\text{ }^{\circ}\text{C/h}$.

$$\begin{aligned} 4 \times (-1) &= -4 \\ 1.5 \times (-1) &= -1.5 \\ -4 + (-1.5) &= -5.5 \end{aligned}$$

$$4 \times (-1) + 1.5 \times (-1) = -5.5$$

$$-5 \div 5 = -1$$

Why are the time periods represented by positive rational numbers?

Why are the rates of temperature decrease represented by negative rational numbers?

Literacy Link

Order of Operations

- Perform operations inside parentheses first.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

Show You Know

A hot-air balloon climbed at 0.8 m/s for 10 s.
It then descended at 0.6 m/s for 6 s.

- What was the overall change in altitude?
- What was the average rate of change in altitude?



Key Ideas

- One way to model the addition of rational numbers is on a number line. One way to model the subtraction of rational numbers is by adding the opposite on a number line.

$$-2.2 - 1.3 = -3.5 \quad \text{or} \quad -2.2 + (-1.3) = -3.5$$

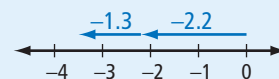
- The product or quotient of two rational numbers with the same signs is positive.

$$-1.2 \times (-1.5) = 1.8 \quad -1.5 \div (-1.2) = 1.25$$

The product or quotient of two rational numbers with different signs is negative.

$$1.2 \times (-1.5) = -1.8 \quad -1.5 \div 1.2 = -1.25$$

- The order of operations for calculations involving rational numbers is the same as for whole numbers, decimals, and integers.
 - Perform operations inside parentheses first.
 - Divide and multiply in order from left to right.
 - Add and subtract in order from left to right.



Check Your Understanding

Communicate the Ideas

- Would you expect the subtraction $-3.5 - (-4.3)$ to give a positive answer or a negative answer? Explain.
 - Evaluate $-3.5 - (-4.3)$.
- How do the values of the following two products compare? Explain your reasoning.
 $2.54 \times (-4.22)$ -2.54×4.22
- Leslie evaluated $-2.2 + 4.6 \times (-0.5)$ as -1.2 . Zack evaluated the same expression as -4.5 . Who was correct? Explain.

Practise

For help with #4 and #5, refer to Example 1 on page 56.

4. Estimate and calculate.

- a) $0.98 + (-2.91)$
- b) $5.46 - 3.16$
- c) $-4.23 + (-5.75)$
- d) $-1.49 - (-6.83)$

5. Calculate.

- a) $9.37 - 11.62$
- b) $-0.512 + 2.385$
- c) $0.675 - (-0.061)$
- d) $-10.95 + (-1.99)$

For help with #6 and #7, refer to Example 2 on page 57.

6. Estimate and calculate.

- a) $2.7 \times (-3.2)$
- b) $-3.25 \div 2.5$
- c) $-5.5 \times (-5.5)$
- d) $-4.37 \div (-0.95)$

7. Calculate. Express your answer to the nearest thousandth, if necessary.

- a) $-2.4(-1.5)$
- b) $8.6 \div 0.9$
- c) $-5.3(4.2)$
- d) $19.5 \div (-16.2)$
- e) $1.12(0.68)$
- f) $-0.55 \div 0.66$

For help with #8 to #11, refer to Example 3 on page 58.

8. Evaluate.

- a) $-2.1 \times 3.2 + 4.3 \times (-1.5)$
- b) $-3.5(4.8 - 5.6)$
- c) $-1.1[2.3 - (-0.5)]$

Literacy Link

In $-1.1[2.3 - (-0.5)]$, square brackets are used for grouping because -0.5 is already in parentheses.

9. Determine each value.

- a) $(4.51 - 5.32)(5.17 - 6.57)$
- b) $2.4 + 1.8 \times 5.7 \div (-2.7)$
- c) $-4.36 + 1.2[2.8 + (-3.5)]$

10. In Regina, Saskatchewan, the average mid-afternoon temperature in January is -12.6°C . The average mid-afternoon temperature in July is 26.1°C . How many degrees colder is Regina in January than in July?

11. One January day in Penticton, British Columbia, the temperature read -6.3°C at 10:00 a.m. and 1.4°C at 3:00 p.m.

- a) What was the change in temperature?
- b) What was the average rate of change in temperature?

Apply

12. A pelican dives vertically from a height of 3.8 m above the water. It then catches a fish 2.3 m underwater.

- a) Write an expression using rational numbers to represent the length of the pelican's dive.
- b) How long is the pelican's dive?

13. A submarine was cruising at a depth of 153 m. It then rose at 4.5 m/min for 15 min.

- a) What was the submarine's depth at the end of this rise?
- b) If the submarine continues to rise at the same rate, how much longer will it take to reach the surface?

14. Saida owned 125 shares of an oil company. One day, the value of each share dropped by $\$0.31$. The next day, the value of each share rose by $\$0.18$. What was the total change in the value of Saida's shares?

Literacy Link

A *share* is one unit of ownership in a corporation.

15. In dry air, the temperature decreases by about $0.65\text{ }^{\circ}\text{C}$ for each 100-m increase in altitude.

- The temperature in Red Deer, Alberta, is $10\text{ }^{\circ}\text{C}$ on a dry day. What is the temperature outside an aircraft 2.8 km above the city?
- The temperature outside an aircraft 1600 m above Red Deer is $-8.5\text{ }^{\circ}\text{C}$. What is the temperature in the city?

16. Bella is more comfortable working with integers than with positive and negative decimal numbers. This is her way of understanding $-4.3 + 2.5$.

-4.3 is $\frac{-43}{10}$ or -43 tenths.

2.5 is $\frac{25}{10}$ or 25 tenths.

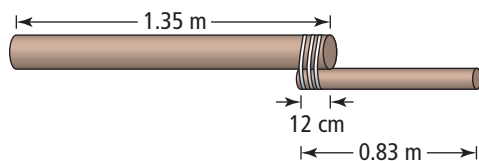
-43 tenths + 25 tenths is -18 tenths.

-18 tenths is $\frac{-18}{10}$ or -1.8 .

So, $-4.3 + 2.5 = -1.8$.

- Use Bella's method to determine $6.1 + (-3.9)$.
- How could you modify Bella's method to determine $1.25 - 3.46$?

17. Two wooden poles measured 1.35 m and 0.83 m in length. To make a new pole, they were attached by overlapping the ends and tying them together. The length of the overlap was 12 cm. What was the total length of the new pole in metres?



18. Determine the mean of each set of numbers. Express your answer to the nearest hundredth, if necessary.

- $0, -4.5, -8.2, 0.4, -7.6, 3.5, -0.2$
- $6.3, -2.2, 14.9, -4.8, -5.3, 1.6$

19. A company made a profit of \$8.6 million in its first year. It lost \$5.9 million in its second year and lost another \$6.3 million in its third year.

- What was the average profit or loss per year over the first three years?
- The company broke even over the first four years. What was its profit or loss in the fourth year?

20. Research to find out the current price of gasoline in Calgary. It is 300 km from Calgary to Edmonton. How much less would it cost to drive this distance in a car with a fuel consumption of 5.9 L/100 km than in a car with a fuel consumption of 9.4 L/100 km? Give your answer in dollars, expressed to the nearest cent.



WWW Web Link

To find out prices of gas in Calgary, go to www.mathlinks9.ca and follow the links.

21. Andrew drove his car 234 km from Dawson to Mayo in Yukon Territory in 3 h. Brian drove his truck along the same route at an average speed of 5 km/h greater than Andrew's average speed. How much less time did Brian take, to the nearest minute?

22. An aircraft was flying at an altitude of 2950 m. It descended for 3 min at 2.5 m/s and then descended for 2.5 min at 2.8 m/s. What was the plane's altitude after the descent?

- 23.** One week in October in Iqaluit, Nunavut, the daily high temperatures were $-4.6\text{ }^{\circ}\text{C}$, $-0.5\text{ }^{\circ}\text{C}$, $1.2\text{ }^{\circ}\text{C}$, $2.3\text{ }^{\circ}\text{C}$, $-1.1\text{ }^{\circ}\text{C}$, $1.5\text{ }^{\circ}\text{C}$, and $-3.0\text{ }^{\circ}\text{C}$. What was the mean daily high temperature that week?

Did You Know?

Iqaluit is the capital of Nunavut. This territory covers almost $\frac{1}{5}$ of the area of Canada but has less than $\frac{1}{1000}$ of the Canadian population. Over $\frac{4}{5}$ of the people who live in Nunavut are Inuit.

Web Link

For more information about Nunavut, go to www.mathlinks9.ca and follow the links.

- 24.** Copy and complete each statement.

- a) $\blacksquare + 1.8 = -3.5$
 b) $-13.3 - \blacksquare = -8.9$
 c) $\blacksquare \times (-4.5) = -9.45$
 d) $-18.5 \div \blacksquare = 7.4$

- 25.** Create a word problem that involves operations with rational numbers in decimal form. Make sure you can solve your problem. Then, have a classmate solve your problem.

Extend

- 26.** Four points, A, B, C, and D, have the following coordinates:

A(0.75, 0.81)
 B(0.75, -0.65)
 C(-1.22 , -0.65)
 D(-1.22 , 0.81)

What is the area of quadrilateral ABCD, to the nearest hundredth of a square unit?

- 27.** The mean of six rational numbers is -4.3 .
- a) What is the sum of the rational numbers?
 b) If five of the numbers each equal -4.5 , what is the sixth number?

- 28.** Evaluate each expression.

a) $3.6 + 2y$, $y = -0.5$
 b) $(m - 1.8)(m + 1.8)$, $m = 1.7$
 c) $\frac{4.5}{q} - \frac{q}{4.5}$, $q = -3.6$

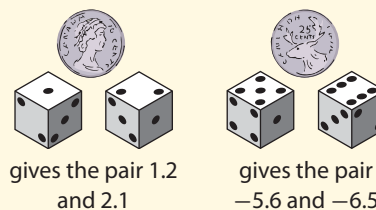
- 29.** Add one pair of parentheses or square brackets to the left side of each equation to make a true statement.

a) $3.5 \times 4.1 - 3.5 - 2.8 = -0.7$
 b) $2.5 + (-4.1) + (-2.3) \times (-1.1) = 4.29$
 c) $-5.5 - (-6.5) \div 2.4 + (-1.1) = -0.5$

Math Link

Play this game with a partner or in a small group. You will need two dice and one coin.

- For each turn, roll the two dice and toss the coin. Then, repeat.
- Create numbers of the form $\blacksquare.\blacksquare$ from the result of rolling the two dice.
- Tossing heads means the rational numbers are positive. Tossing tails means the rational numbers are negative.
- Record the two pairs of numbers.
- Choose one number from each pair so that the sum of the chosen numbers is as close as possible to zero. Record the sum of the chosen numbers.
- In each round, the player with the sum one closest to zero wins two points. If there is a tie, each tied player wins one point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.



The four possible sums are
 $1.2 + (-5.6)$ $1.2 + (-6.5)$
 $2.1 + (-5.6)$ $2.1 + (-6.5)$
 Estimation shows that $2.1 + (-5.6)$ is closest to zero, so calculate this sum.
 $2.1 + (-5.6) = -3.5$