# 2.1 Comparing and Ordering Rational Numbers

## Focus on...

After this lesson, you will be able to...

- compare and order rational numbers
- identify a rational number between two given rational numbers

Cape Dorset, Nunavut

The percent of Canadians who live in rural areas has been decreasing since 1867. At that time, about 80% of Canadians lived in rural areas. Today, about 80% of Canadians live in urban areas, mostly in cities. The table shows changes in the percent of Canadians living in urban and rural areas over four decades.

**Did You Know?**

An urban area has a population of 1000 or more. In urban areas, 400 or more people live in each square kilometre. Areas that are not urban are called rural. What type of area do you live in?

| Decade | Change in the Percent of Canadians in Urban Areas (%) | Change in the Percent of Canadians in Rural Areas (%) |
|---|---|---|
| 1966–1976 | +1.9 | −1.9 |
| 1976–1986 | +1.0 | −1.0 |
| 1986–1996 | +1.4 | −1.4 |
| 1996–2006 | +2.3 | −2.3 |

How can you tell that some changes in the table are increases and others are decreases?

## Explore Rational Numbers

**1.** How are the **rational numbers** in the table on page 46 related. Explain your reasoning.

**2. a)** Choose a rational number in decimal form. Identify its opposite. How do you know these are opposite rational numbers?

**b)** Choose a rational number in fraction form. Identify its opposite.

**c)** Identify another pair of opposite rational numbers.

**3. a)** Identify equivalent rational numbers from the following list.

$\frac{12}{4}$ $\frac{-8}{4}$ $\frac{-9}{-3}$ $-\frac{4}{2}$ $\frac{4}{-2}$ $\frac{12}{3}$ $-\left(\frac{4}{-1}\right)$ $-\left(\frac{-4}{-2}\right)$

**b)** Choose a rational number in fraction form that is not equivalent to any of the rational numbers in part a). Challenge a classmate to write four rational numbers that are equivalent to your chosen number.

### Reflect and Check

**4. a)** How can you identify opposite rational numbers?

**b)** How can you identify equivalent rational numbers?

**5. a)** Predict what you think the change in the percent of Canadians in urban areas from 2006 to 2016 might be. Justify your prediction.

**b)** What would you expect the change in the percent of Canadians in rural areas to be for that decade? Explain.

**rational number**

- a number that can be expressed as $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$
- examples include $-4$, $3.5$, $-\frac{1}{2}$, $1\frac{3}{4}$, and $0$

**Literacy Link**

When numbers are *equivalent*, they have the same value. $\frac{24}{-4}$, $\frac{-18}{3}$, $-\frac{12}{2}$, and $-\left(\frac{-6}{-1}\right)$ are all equivalent. They all represent the same rational number. What is it?

Vancouver, British Columbia

## Link the Ideas

### Example 1: Compare and Order Rational Numbers

Compare and order the following rational numbers.

$-1.2 \quad \frac{4}{5} \quad \frac{7}{8} \quad -0.\overline{5} \quad -\frac{7}{8}$

#### *Solution*

You can estimate the order.

$-1.2$ is a little less than $-1$.

$\frac{4}{5}$ is a little less than 1.

$\frac{7}{8}$ is a little less than 1.

$-0.\overline{5}$ is a little less than $-0.5$.

$-\frac{7}{8}$ is a little more than $-1$.

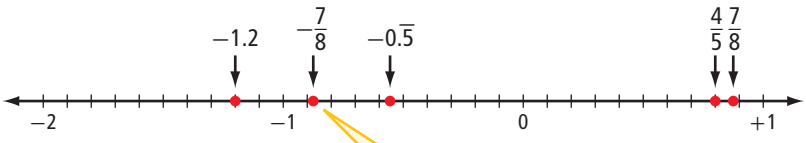
An estimate of the order from least to greatest is $-1.2, -\frac{7}{8}, -0.\overline{5}, \frac{4}{5}, \frac{7}{8}$.

Express all the numbers in the same form.
You can write the numbers in decimal form.

**Strategies**
**Draw a Diagram**

$-1.2 \quad \frac{4}{5} = 0.8 \quad \frac{7}{8} = 0.875 \quad -0.\overline{5} = -0.555\ldots \quad -\frac{7}{8} = -0.875$

Place the numbers on a number line.




What number is the opposite of $-\frac{7}{8}$? How does the position of that number on the number line compare with the position of $-\frac{7}{8}$?

The numbers in ascending order are $-1.2, -\frac{7}{8}, -0.\overline{5}, \frac{4}{5}$, and $\frac{7}{8}$.

The numbers in descending order are $\frac{7}{8}, \frac{4}{5}, -0.\overline{5}, -\frac{7}{8}$, and $-1.2$.

**Show You Know**

Compare the following rational numbers. Write them in ascending order and descending order.

$0.\overline{3} \quad -0.6 \quad -\frac{3}{4} \quad 1\frac{1}{5} \quad -1$

## Example 2: Compare Rational Numbers

Which fraction is greater, $-\frac{3}{4}$ or $-\frac{2}{3}$?

### Solution

*Method 1: Use Equivalent Fractions*

You can express the fractions as equivalent fractions with a common denominator.

A common denominator of the two fractions is 12.

How do you know 12 is a common denominator?

$$-\frac{3}{4} \overset{\times 3}{=} -\frac{9}{12} \qquad -\frac{2}{3} \overset{\times 4}{=} -\frac{8}{12}$$

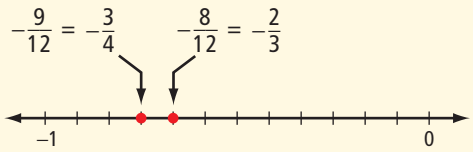
When the denominators are the same, compare the numerators.

$-\frac{9}{12} = \frac{-9}{12}$ $\quad -\frac{8}{12} = \frac{-8}{12}$

$\frac{-8}{12} > \frac{-9}{12}$, because $-8 > -9$.

$-\frac{2}{3}$ is the greater fraction.

How does the number line show the comparison?




**Literacy Link**

The quotient of two integers with unlike signs is negative. This means that

$-\frac{9}{12} = \frac{-9}{12} = \frac{9}{-12}$

and

$-\frac{8}{12} = \frac{-8}{12} = \frac{8}{-12}$.

*Method 2: Use Decimals*

You can also compare by writing the fractions as decimal numbers.

$-\frac{3}{4} = -0.75$

$-\frac{2}{3} = -0.\overline{6}$

$-0.\overline{6} > -0.75$

$-\frac{2}{3}$ is the greater fraction.

**Show You Know**

Which fraction is smaller, $-\frac{7}{10}$ or $-\frac{3}{5}$?

**WWW Web Link**

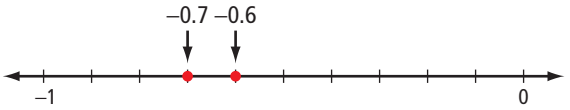
For practice comparing and ordering rational numbers, go to www.mathlinks9.ca and follow the links.

## Example 3: Identify a Rational Number Between Two Given Rational Numbers

Identify a fraction between $-0.6$ and $-0.7$.

### *Solution*

You can first identify a decimal number between $-0.6$ and $-0.7$, using a number line.




You can also change $-0.6$ and $-0.7$ into fraction form. What would the number line look like?

One decimal number between $-0.6$ and $-0.7$ is $-0.65$.

Convert the decimal to a fraction. $-0.65 = -\frac{65}{100}$

A fraction between $-0.6$ and $-0.7$ is $-\frac{65}{100}$.

What is another way to express $-\frac{65}{100}$ as a fraction?

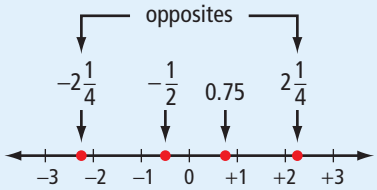
**Strategies**
**Draw a Diagram**

**Show You Know**

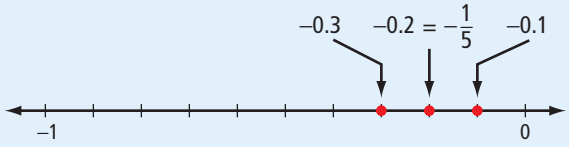
Identify a fraction between $-2.4$ and $-2.5$.

## Key Ideas

- Rational numbers can be positive, negative, or zero. They include integers, positive and negative fractions, mixed numbers, and decimal numbers.
  Examples: $-6$, $15$, $\frac{3}{4}$, $-1\frac{2}{3}$, $3.9$, $-2.\overline{3}$
- Equivalent fractions represent the same rational number.
  $-\frac{5}{2}$, $\frac{-5}{2}$, $\frac{10}{-4}$, and $-\left(\frac{-10}{-4}\right)$ all represent $-2\frac{1}{2}$ or $-2.5$.
- One strategy for comparing and ordering rational numbers is to use a number line.
  - On a horizontal number line, a larger rational number is to the right of a smaller rational number.
  - Opposite rational numbers are the same distance in opposite directions from zero.



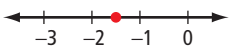

- You can compare fractions with the same denominator by comparing the numerators.
  $\frac{-7}{10} < \frac{-6}{10}$, because $-7 < -6$.
- One strategy for identifying a rational number between two given rational numbers is to use a number line.
  A rational number in fraction form between $-0.3$ and $-0.1$ is $\frac{-1}{5}$.



# Check Your Understanding

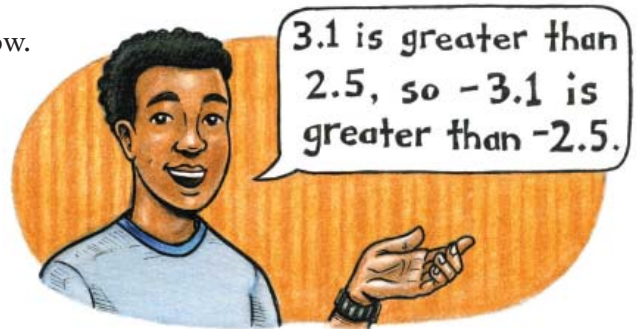
## Communicate the Ideas

**1.** Laura placed $-2\frac{1}{2}$ incorrectly on a number line, as shown.




How could you use the idea of opposites to show Laura how to plot $-2\frac{1}{2}$ correctly?

**2.** Is Dominic correct? Show how you know.




**3.** Tomas and Roxanne were comparing $-0.9$ and $-\frac{7}{8}$. Tomas wrote $-0.9$ as a fraction, and then he compared the two fractions. Roxanne wrote $-\frac{7}{8}$ as a decimal, and then she compared the two decimals.

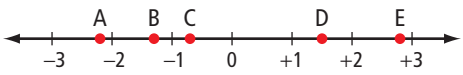
**a)** Which method do you prefer? Explain.

**b)** Which is greater, $-0.9$ or $-\frac{7}{8}$? Explain how you know.

## Practise

***For help with #4 to #9, refer to Example 1 on page 48.***

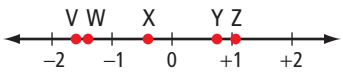
**4.** Match each rational number to a point on the number line.




**a)** $\frac{3}{2}$ **b)** $-0.7$ **c)** $-2\frac{1}{5}$

**d)** $\frac{14}{5}$ **e)** $-1\frac{1}{3}$

**5.** Which point on the number line matches each rational number?




**a)** $-1\frac{2}{5}$ **b)** $\frac{3}{4}$ **c)** $1\frac{1}{20}$

**d)** $-1\frac{3}{5}$ **e)** $-0.\overline{4}$

**6.** Place each number and its opposite on a number line.

**a)** $\frac{8}{9}$ **b)** $-1.2$ **c)** $2\frac{1}{10}$ **d)** $-\frac{11}{3}$

**7.** What is the opposite of each rational number?

**a)** $-4.\overline{1}$ **b)** $\frac{4}{5}$ **c)** $-5\frac{3}{4}$ **d)** $\frac{9}{8}$

**8.** Compare $1\frac{5}{6}$, $-1\frac{2}{3}$, $-0.1$, $1.9$, and $-\frac{1}{5}$. Write the numbers in ascending order.

**9.** Compare $-\frac{3}{8}$, $1.\overline{8}$, $\frac{9}{5}$, $-\frac{1}{2}$, and $-1$. Write the numbers in descending order.

*For help with #10 to #13, refer to Example 2 on page 49.*

**10.** Express each fraction as an equivalent fraction.

**a)** $-\frac{2}{5}$ **b)** $\frac{10}{6}$

**c)** $-\frac{9}{12}$ **d)** $\frac{-4}{3}$

**11.** Write each rational number as an equivalent fraction.

**a)** $\frac{-1}{3}$ **b)** $\frac{-4}{-5}$

**c)** $-\left(\frac{-5}{-4}\right)$ **d)** $\frac{7}{-2}$

**12.** Which value in each pair is greater?

**a)** $\frac{1}{3}, -\frac{2}{3}$ **b)** $-\frac{9}{10}, \frac{7}{10}$

**c)** $-\frac{1}{2}, -\frac{3}{5}$ **d)** $-2\frac{1}{8}, -2\frac{1}{4}$

**13.** Which value in each pair is smaller?

**a)** $\frac{4}{7}, \frac{2}{3}$ **b)** $-\frac{4}{3}, -\frac{5}{3}$

**c)** $-\frac{7}{10}, -\frac{3}{5}$ **d)** $-1\frac{3}{4}, -1\frac{4}{5}$

*For help with #14 to #17, refer to Example 3 on page 50.*

**14.** Identify a decimal number between each of the following pairs of rational numbers.

**a)** $\frac{3}{5}, \frac{4}{5}$ **b)** $-\frac{1}{2}, -\frac{5}{8}$

**c)** $-\frac{5}{6}, 1$ **d)** $-\frac{17}{20}, -\frac{4}{5}$

**15.** What is a decimal number between each of the following pairs of rational numbers?

**a)** $1\frac{1}{2}, 1\frac{7}{10}$ **b)** $-2\frac{2}{3}, -2\frac{1}{3}$

**c)** $1\frac{3}{5}, -1\frac{7}{10}$ **d)** $-3\frac{1}{100}, -3\frac{1}{50}$

**16.** Identify a fraction between each of the following pairs of rational numbers.

**a)** 0.2, 0.3 **b)** 0, −0.1

**c)** −0.74, −0.76 **d)** −0.52, −0.53

**17.** Identify a mixed number between each of the following pairs of rational numbers.

**a)** 1.7, 1.9 **b)** −0.5, 1.5

**c)** −3.3, −3.4 **d)** −2.01, −2.03

## Apply

**18.** Use a rational number to represent each quantity. Explain your reasoning.

**a)** a temperature increase of 8.2 °C

**b)** growth of 2.9 cm


**c)** 3.5 m below sea level

**d)** earnings of $32.50

**e)** 14.2 °C below freezing

**19.** The table includes the melting points and boiling points of six elements known as the noble gases.

| Noble Gas | Melting Point (°C) | Boiling Point (°C) |
|---|---|---|
| Argon | −189.2 | −185.7 |
| Helium | −272.2 | −268.6 |
| Neon | −248.67 | −245.92 |
| Krypton | −156.6 | −152.3 |
| Radon | −71.0 | −61.8 |
| Xenon | −111.9 | −107.1 |

**a)** Which noble gases have a melting point that is less than the melting point of argon?

**b)** Which noble gases have a boiling point that is greater than the boiling point of krypton?

**c)** Arrange the melting points in ascending order.

**d)** Arrange the boiling points in descending order.

**Science Link**

For many years, the noble gases were known as the *inert gases*. Most chemists thought that these gases would not react with other chemicals. In 1962, Neil Bartlett, a chemist at the University of British Columbia, proved them wrong.

**WWW Web Link**

To learn more about Neil Bartlett and to research Canadian scientific discoveries, go to www.mathlinks9.ca and follow the links.

**20. a)** Kwasi said that he ignored the fractions when he decided that $-2\frac{1}{5}$ is smaller than $-1\frac{9}{10}$. Explain his thinking.

**b)** Naomi said that she ignored the integer $-1$ when she decided that $-1\frac{1}{4}$ is greater than $-1\frac{2}{7}$. Explain her thinking.

**21.** The table shows the average early-morning temperature for seven communities in May.

| Community | Average Early-Morning Temperature (°C) |
|---|---|
| Churchill, Manitoba | −5.1 |
| Regina, Saskatchewan | 3.9 |
| Edmonton, Alberta | 5.4 |
| Penticton, British Columbia | 6.1 |
| Yellowknife, Northwest Territories | −0.1 |
| Whitehorse, Yukon Territory | 0.6 |
| Resolute, Nunavut | −14.1 |

**a)** Write the temperatures in descending order.

**b)** Which community has an average temperature between the values for Whitehorse and Churchill?

**22.** Replace each ■ with $>$, $<$, or $=$ to make each statement true.

**a)** $\frac{-9}{6} \blacksquare \frac{3}{-2}$

**b)** $-\frac{3}{5} \blacksquare -0.\overline{6}$

**c)** $-1\frac{3}{10} \blacksquare -\left(\frac{-13}{-10}\right)$

**d)** $-3.25 \blacksquare -3\frac{1}{5}$

**e)** $-\frac{8}{12} \blacksquare -\frac{11}{15}$

**f)** $-2\frac{5}{6} \blacksquare -2\frac{7}{8}$

**23.** Is zero a rational number? Explain.

**24.** Give an example of a fraction in lowest terms that satisfies the following conditions.

**a)** greater than 0, with the denominator greater than the numerator

**b)** between 0 and −1, with the denominator less than the numerator

**c)** less than −2, with the numerator less than the denominator

**d)** between −1.2 and −1.3, with the numerator greater than the denominator

**25.** Which integers are between $\frac{11}{5}$ and $\frac{15}{-4}$?

**26.** Which number in each pair is greater? Explain each answer.

**a)** 0.4 and 0.44

**b)** $0.\overline{3}$ and 0.33

**c)** $-0.7$ and $-0.77$

**d)** $-0.66$ and $-0.\overline{6}$

**27.** Identify the fractions that are between 0 and $-2$ and that have 3 as the denominator.

## Extend

**28.** How many rational numbers are between $\frac{2}{3}$ and $0.\overline{6}$? Explain.

**29.** Replace each ■ with an integer to make each statement true. In each case, is more than one answer possible? Explain.

**a)** $\blacksquare.5 < -1.9$

**b)** $\frac{\blacksquare}{-4} = -2\frac{1}{4}$

**c)** $\frac{-3}{\blacksquare} = -\frac{-15}{5}$

**d)** $-1.5\blacksquare 2 > -1.512$

**e)** $-\frac{3}{4} < -0.7\blacksquare$

**f)** $-5\frac{1}{2} > \frac{11}{\blacksquare}$

**g)** $-2\frac{3}{5} = \frac{\blacksquare}{10}$

**h)** $\frac{8}{\blacksquare} < -\frac{2}{3}$

**30.** Determine the value of $x$.

**a)** $\frac{4}{-5} = \frac{x}{-10}$

**b)** $\frac{x}{3} = \frac{6}{-9}$

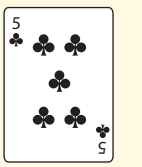
**c)** $\frac{5}{x} = -\frac{20}{12}$

**d)** $\frac{-6}{-5} = \frac{30}{x}$

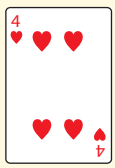
## Math Link

Play the following game with a partner or in a small group. You will need one deck of playing cards.

- Remove the jokers, aces, and face cards from the deck.
- Red cards represent positive integers. Black cards represent negative integers.
- In each round, the dealer shuffles the cards and deals two cards to each player.
- Use your two cards to make a fraction that is as close as possible to zero.
- In each round, the player with the fraction closest to zero wins two points. If there is a tie, each tied player wins a point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.



represents $-5$



represents 4

With a five of clubs and a four of hearts, you can make $\frac{4}{-5}$ or $\frac{-5}{4}$. Choose $\frac{4}{-5}$ because it is closer to zero.