10.3 Tangents to a Circle

Focus on...

After this lesson, you will be able to...

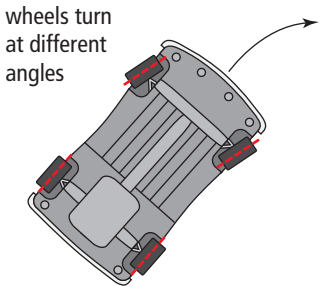
- **relate tangent lines to the radius of the circle.**

When a car turns, the wheels are at different angles in relation to the car. Each wheel is turning through its own circle. What is the relationship between the four circles where the tires turn?

Materials

- Turning Circle diagram
- protractor
- ruler

Explore Circles and Their Tangents



1. Find the midpoint of each line segment that represents a tire.

2. Draw a perpendicular line from each midpoint toward the inside of the turning circle.

Reflect and Check

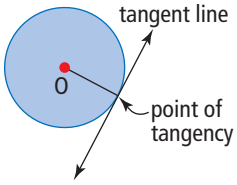
3. What do you notice about the intersection of these perpendicular lines?

4. **a)** Each wheel of a car travels through a different circular path. What do these circles have in common?

   b) Based on your observations, what is the measure of the angle between a tangent to a circle and the radius at the point of tangency?

tangent (to a circle)

- a line that touches a circle at exactly one point
- the point where the line touches the circle is called the point of tangency



WWW Web Link

You may wish to explore these geometric properties on a computer. Go to www.mathlinks9.ca and follow the links.

Link the Ideas

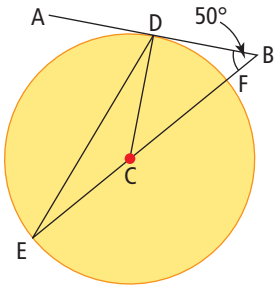
You can use properties of tangents to a circle to solve problems.

Tangent to a Circle
A tangent to a circle is perpendicular to the radius at the point of tangency.

Tangent Chord Relationship
A chord drawn perpendicular to a tangent at the point of tangency contains the centre of the circle, and is a diameter.

Example 1: Determine Angle Measures in a Circle With a Tangent Line

In the diagram shown, AB is tangent to the circle at point D, BE contains the diameter FE, and $\angle ABE = 50°$.



a) What is the measure of $\angle BDC$? Justify your answer.
b) What is the measure of central angle $\angle DCE$? Explain your reasoning.
c) What type of triangle is $\triangle CDE$? Justify your answer.
d) What is the measure of $\angle DEC$? Explain your reasoning.

Solution

a) Since AB is tangent to the circle at point D, then radius CD is perpendicular to line segment AB. Therefore, $\angle BDC = 90°$.

b) The sum of the angles in a triangle is 180°.
In $\triangle BCD$, $\angle DCB = 180° - 90° - 50°$
$\angle DCB = 40°$

Since $\angle DCE$ and $\angle DCB$ form a straight line, they are supplementary.
$\angle DCE + \angle DCB = 180°$
$\angle DCE + 40° = 180°$
$\angle DCE = 180° - 40°$
$\angle DCE = 140°$

c) Triangle CDE is an isosceles triangle because CD and CE are radii of the circle and radii are equal in length.

Literacy Link
Supplementary angles add to 180°.

d) *Method 1: Use Angles in a Triangle*

The sum of the angles in a triangle is 180°. ∠DCE = 140°. Since △CDE is an isosceles triangle, then the angles opposite the equal sides are equal. $\angle DEC = \frac{1}{2} \times 40°$ or 20°.

∠DEC = 20°.

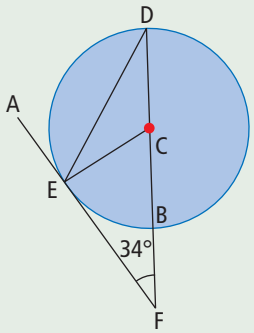
Method 2: Use Inscribed Angles

∠DEF is the same as ∠DEC because the points F and C lie on the same line. This is an inscribed angle subtended by the same arc as the central angle, ∠DCF. Since an inscribed angle is one half the measure of a central angle subtended by the same arc, then $\angle DEC = \frac{1}{2} \times 40°$ or 20°.

∠DEC = 20°

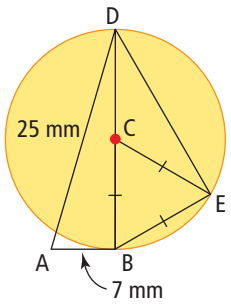
Show You Know

Line segment AF is tangent to the circle at point E. Line segment DF contains the diameter DB, and ∠CFE = 34°. What are the measures of angles ∠CEF, ∠ECF, and ∠EDF? Explain your reasoning.



Example 2: Use the Tangent Chord Relationship

In the diagram, AB is tangent to the circle at point B. BD is a diameter of the circle. AB = 7 mm, AD = 25 mm, and △BCE is an equilateral triangle.



a) What is the length of diameter BD? Justify your answer.
b) What is the length of chord BE? Explain your reasoning.
c) What is the measure of the inscribed angle ∠BED?
d) What is the length of chord DE? Justify your answer and express your answer to the nearest millimetre.

Solution

Strategies
Organize, Analyse, Solve

a) Diameter BD is perpendicular to tangent AB because B is the point of tangency on the circle. Therefore, $\angle ABD = 90°$ and $\triangle ABD$ is a right triangle.

Use the Pythagorean relationship in $\triangle ABD$.

$$\begin{aligned} AB^2 + BD^2 &= AD^2 \\ 7^2 + BD^2 &= 25^2 \\ 49 + BD^2 &= 625 \\ BD^2 &= 576 \\ BD &= \sqrt{576} \\ BD &= 24 \end{aligned}$$

The length of diameter BD is 24 mm.

b) BC and CE are radii of the circle. Since $\triangle BCE$ is an equilateral triangle, side BE is equal the length of the radius, or one half of the diameter.

$\frac{1}{2}(24) = 12$

The length of chord BE is 12 mm

c) The inscribed angle $\angle BED$ is subtended by a diameter, so it is a right angle. $\angle BED = 90°$.

The inscribed angle $\angle BED = 90°$.

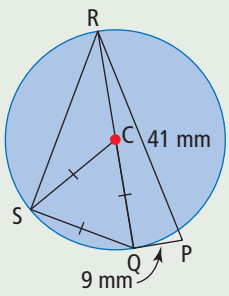
d) Use the Pythagorean relationship in $\triangle BDE$.

$$\begin{aligned} BE^2 + DE^2 &= BD^2 \\ 12^2 + DE^2 &= 24^2 \\ 144 + DE^2 &= 576 \\ DE^2 &= 576 - 144 \\ DE^2 &= 432 \\ DE &= \sqrt{432} \\ DE &\approx 21 \end{aligned}$$

The length of chord DE is 21 mm, to the nearest millimetre.

Show You Know

In the diagram shown, PQ is tangent to the circle at point Q. QR is a diameter of the circle. Line segment PQ = 9 mm, PR = 41 mm, and $\triangle QCS$ is an equilateral triangle.



a) What is the length of diameter QR? Justify your answer.

b) What is the length of chord QS? Explain your reasoning.

c) What is the length of chord RS? Justify your answer and express your answer to the nearest millimetre.

Science Link

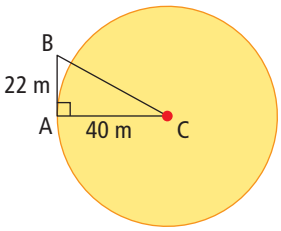
An object that is moving in a circular path will move in a straight line tangent to that circle if the force pulling the object toward the centre is suddenly removed. This force is known as *centripetal force*.

Example 3: Solve Problems With Tangents to Circles

A speed skater is practising on a circular track with a radius of 40 m. He falls and slides off the track in a line tangent to the circle. If he slides 22 m, how far is he from the centre of the rink? Express your answer to the nearest tenth of a metre. Include a diagram in your explanation.

Solution

In the diagram, the speed skater fell at point A and slid to point B.



Since the line segment AB is tangent to the circle, then it will be perpendicular to radius AC. The Pythagorean relationship can be used to calculate the distance BC, which represents how far the speed skater is from the centre of the rink.

$$BC^2 = AB^2 + AC^2$$
$$BC^2 = 22^2 + 40^2$$
$$BC^2 = 484 + 1600$$
$$BC^2 = 2084$$
$$BC = \sqrt{2084}$$
$$BC \approx 45.7$$

After sliding 22 m, the speed skater is approximately 45.7 m from the centre of the rink.

Sports Link

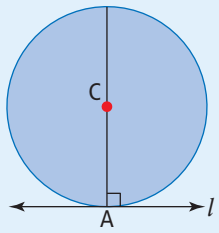
Jeremy Wotherspoon from Red Deer, Alberta, is one of Canada's best speed skaters. He has set several records at the 500-m distance.

Show You Know

Callan is attempting to land his model airplane when the wire breaks just before touchdown. If the length of the control wire is 10 m and the plane stops at a location 74 m from Callan, how far does the plane travel after the wire breaks. Express your answer to the nearest tenth of a metre.

Key Ideas

- A line that touches a circle at exactly one point is tangent to the circle.
- Point A is known as the point of tangency.
- A line l that is tangent to a circle at point A is perpendicular to the radius AC.
- A chord drawn perpendicular to a tangent line at the point of tangency contains the centre of the circle, and is a diameter.

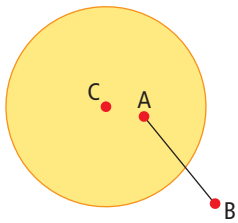


Check Your Understanding

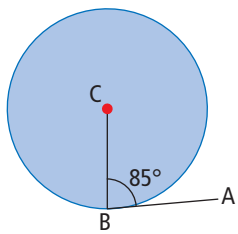
Communicate the Ideas

1. Raven and Elliott are discussing the diagram shown.

Elliott claims that line segment AB is a tangent to the circle because it touches the circle in one place. Raven disagrees. Who is correct, and why?



2. If BC is a radius of the circle, is AB tangent to the circle? Explain how you know.

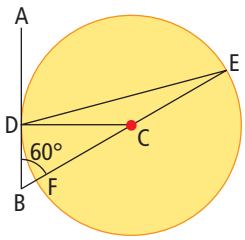


Practise

For help with #3 and #4, refer to Example 1 on pages 395–396.

3. In the diagram, AB is tangent to the circle at point D, BE contains the diameter EF, and $\angle ABE = 60°$.

Explain your reasoning when answering each of the following questions.



a) What is the measure of $\angle BDC$?

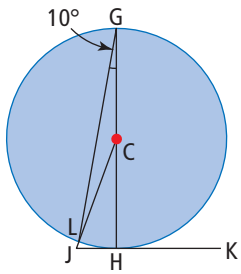
b) What is the measure of central angle $\angle DCE$?

c) What type of triangle is $\triangle CDE$?

d) What is the measure of $\angle DEC$?

4. Line segment JK is tangent to the circle at point H. GH is a diameter and $\angle CGL = 10°$.

Justify your answers to the following questions.



a) What type of triangle is $\triangle CGL$?

b) What is the measure of $\angle GCL$?

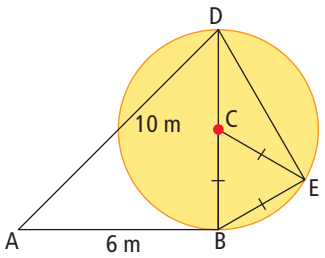
c) What is the measure of $\angle JCH$?

d) What is the measure of $\angle JHG$?

e) What is the measure of $\angle CJK$?

For help with #5 and #6, refer to Example 2 on pages 396–397.

5. In the diagram, AB is tangent to the circle at point B. BD is a diameter of the circle, AB = 6 m, AD = 10 m, and △BCE is an equilateral triangle.



Justify your answers to the following questions.

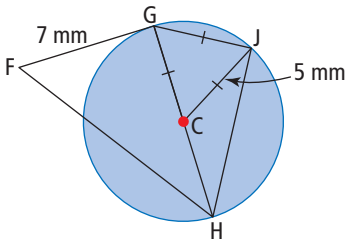
a) What is the length of the diameter BD?

b) What is the length of chord BE?

c) What is the measure of the inscribed angle ∠BED?

d) What is the length of chord DE, to the nearest metre?

6. In the diagram, FG is tangent to the circle at point G. GH is a diameter, CJ = 5 mm, FG = 7 mm, and △CGJ is an equilateral triangle.



a) What is the length of the diameter? Justify your answer.

b) Is △GHJ a right triangle? Justify your answer.

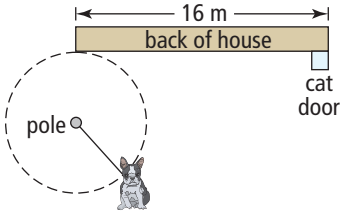
c) What is the length of chord HJ? Explain your reasoning. Express your answer to the nearest tenth of a millimetre.

d) What is the measure of angle ∠FGH? Justify your answer.

e) What is the length of FH? Explain your reasoning. Express your answer to the nearest tenth of a millimetre.

For help with #7, refer to Example 3 on page 398.

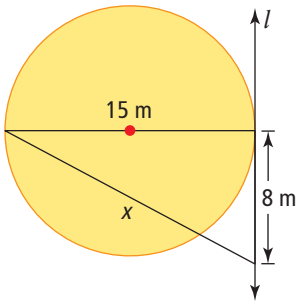
7. A dog is tied on a leash to the clothesline pole in the backyard. The leash is 5 m long and the pole is a perpendicular distance of 5 m from the edge of the house. What is the distance from the pole to the cat door? How close to the cat door can the dog get? Express your answers to the nearest tenth of a metre.



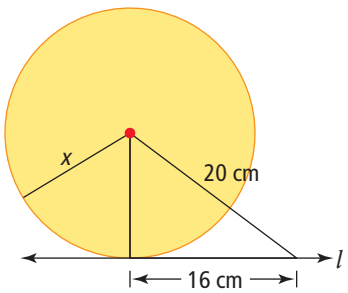
Apply

8. Find the length of x in each diagram. Line l is tangent to the circle. Express your answer to the nearest tenth, where necessary.

a)

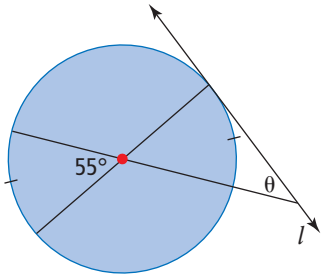


b)

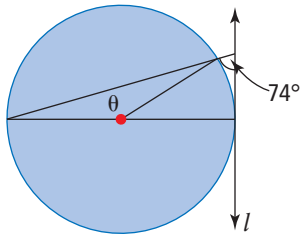


9. Find the measure of the angle θ in each diagram. Line l is tangent to the circle.

a)



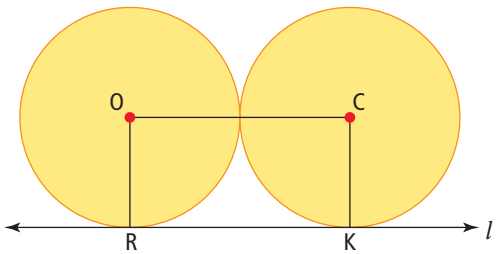
b)



Literacy Link

The Greek letter θ is *theta*. It is often used to indicate the measure of an unknown angle.

10. Both circles are identical in size. They are tangent to each other and to line l.

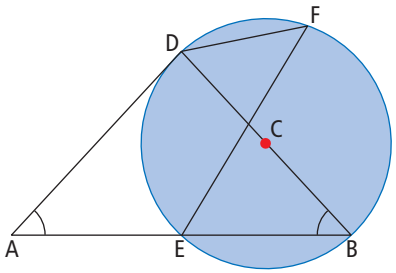


a) What type of quadrilateral is ROCK? Explain your reasoning.

b) If the radius of each circle is 5 cm, what is the perimeter of ROCK?

11. Line segment AB is tangent to a circle at point A. The diameter AD of the circle is 7.3 cm. If the length of AB is 4.2 cm, determine the length of BD. Include a diagram in your solution. Express your answer to the nearest tenth of a centimetre.

12. In the diagram, △ABD is an isosceles triangle. AD is a tangent to the circle at point D, and BD is a diameter of the circle.



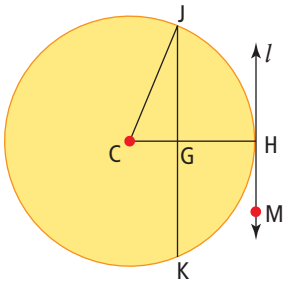
Justify your answers for each question.

a) What is the measure of ∠ADB?

b) What is the measure of ∠DBE?

c) What is the measure of ∠DFE?

13. Answer each question, given the following information.



- The line l is tangent to the circle at point H.
- The line l is parallel to the chord JK.
- The radius of the circle measures 9.1 cm.
- The chord JK measures 17 cm.

Explain your reasoning for each answer.

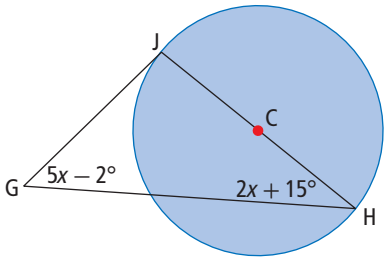
a) What is the measure of ∠CHM?

b) What is the measure of ∠CGJ?

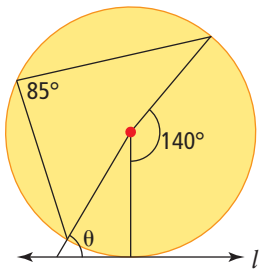
c) What is the length of JG?

d) What is the length of CG? Express your answer to the nearest tenth of a centimetre.

14. If JG is a tangent to the circle, what is the value of x and and the measure of $\angle JGH$?

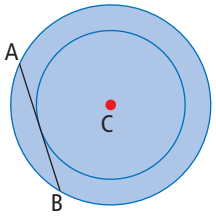


15. Line l is tangent to the circle as shown. Use properties of inscribed and central angles to find the value of angle θ. Explain your reasoning.

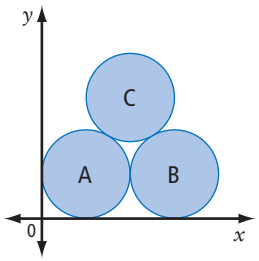


16. The aerial picture represents farmland. The circular green areas represent fields that are watered using a centre-pivot watering system. Design a question and solution using the relationship between tangents and radii of circles.

17. Two concentric circles have their centres at point C. The radius of the smaller circle is 8 cm. The length of chord AB is 26 cm and is tangent to the smaller circle. What is the circumference of the larger circle? Express your answer to the nearest centimetre.

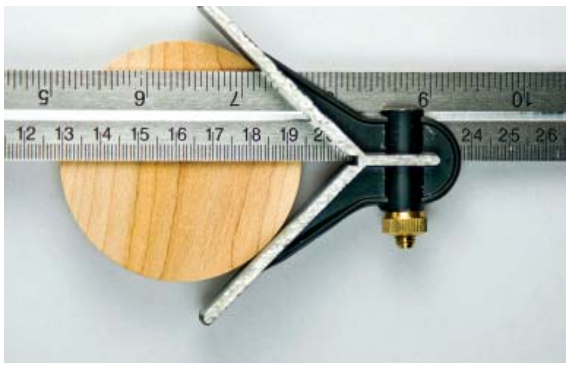


18. Three congruent circles are tangent to one another as shown. Circle A is tangent to both the x-axis and the y-axis. Circle B is tangent to the x-axis. The centre of circle A has coordinates (2, 2). What are the coordinates of the centres of circles B and C?

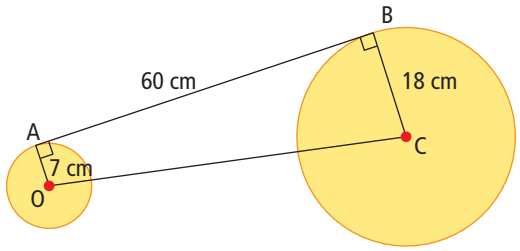


Extend

19. A steel centre square is used in woodwork to locate the centre of a wooden cylinder. Sketch the picture in your notebook and identify the edge(s) that most closely resemble a tangent to a circle. How do you think the centre square is used to locate the centre of the cylinder?

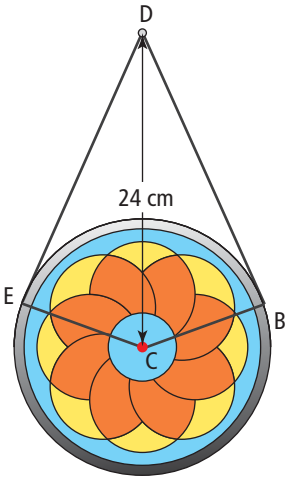


20. Two poles with radii of 18 cm and 7 cm are connected by a single metal band joining their centres and points of their outer edges. This is shown below. Determine the length of the metal band that is needed, if AB is tangent to both poles.



21. A rubber ball with a diameter of 6 cm is found on a frozen pond with only 2 cm sticking above the ice surface at its highest point. What is the circumference of the circle where the ball touches the ice surface? Express your answer to the nearest tenth of a centimetre.

22. A length of chain is attached to a suncatcher with a diameter of 20 cm. The chain is attached at points E and B such that the segments BD and ED are tangent to the circle. What is the total length of chain needed to hang the suncatcher on a nail at point D? Show your reasoning.



Math Link

a) Design a piece of art or a company logo using at least one circle. Incorporate at least one tangent. Remember that two circles can be tangent to each other.

b) Determine the measures of any chords, radii, diameters, or tangent lines in your design.

Tech Link

Tangents to a Circle

In this activity, you will use dynamic geometry software to explore tangent lines to a circle. To use this activity, go to www.mathlinks9.ca and follow the links.

Explore

1. What is the measure of ∠BAC?

2. a) Describe what happens to the measure of ∠BAC as you drag point A to different locations on the circle.

b) What conclusion can you make?