10.2 Exploring Chord Properties

Focus on...

After this lesson, you will be able to...

- describe the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord

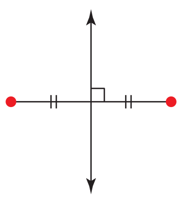
An archeologist found an edge piece of a broken Aztec medallion. If she assumes it is circular, how might she determine the circumference of the whole medallion?

Materials

- compass
- tracing paper
- ruler

Literacy Link

A *perpendicular bisector* passes through the midpoint of a line segment at 90°.



Explore Chords in a Circle

1. Construct a large circle on tracing paper and draw two different chords.

2. Construct the perpendicular bisector of each chord.

What methods could you use to do this construction?

3. Label the point inside the circle where the two perpendicular bisectors intersect.

4. Share your construction method with another classmate.

Reflect and Check

5. a) What do you notice about the point of intersection of the two perpendicular bisectors in step 3?

   b) Do you think that this will be true for any chord and any circle? How could you test your prediction?

6. How could the archeologist use perpendicular bisectors to determine the circumference of the Aztec medallion?

WWW Web Link

You may wish to explore these geometric properties on a computer. Go to www.mathlinks9.ca and follow the links.

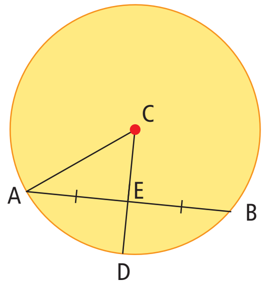
Link the Ideas

You can use properties related to chords in a circle to solve problems.

Perpendicular Bisector of a Chord
A line that passes through the centre of a circle and is perpendicular to a chord bisects the chord.

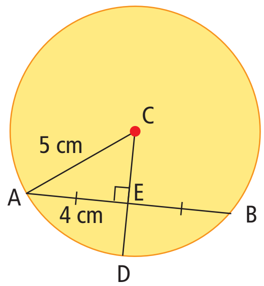
Example 1: Bisect a Chord With a Radius

Radius CD bisects chord AB. Chord AB measures 8 cm. The radius of the circle is 5 cm. What is the length of line segment CE? Justify your solution.



Solution

Since CD is a radius that bisects the chord AB, then CD is perpendicular to AB and ∠AEC = 90°.



Strategies
Organize, Analyse, Solve

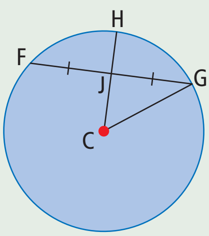
The length of AE is 4 cm because CD bisects the 8-cm chord AB. The radius AC is 5 cm. Using the Pythagorean relationship in △ACE,

$$CE^2 + AE^2 = AC^2$$
$$CE^2 + 4^2 = 5^2$$
$$CE^2 + 16 = 25$$
$$CE^2 = 9$$
$$CE = \sqrt{9}$$
$$CE = 3$$

Therefore, CE measures 3 cm. This is the shortest distance from the chord AB to the centre of the circle.

Show You Know

Radius CH bisects chord FG. Chord FG measures 12 cm. The radius of the circle measures 10 cm. What is the length of CJ?

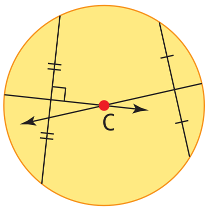


Example 2: Use Chord Properties to Solve Problems

Louise would like to drill a hole in the centre of a circular table in order to insert a sun umbrella. Use a diagram to explain how she could locate the centre.

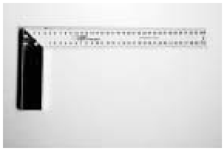
Solution

Draw two chords. Locate the midpoint of each chord. Use a carpenter's square to draw the perpendicular bisectors of each chord. Locate the point of intersection of the two perpendicular bisectors. The point of intersection is the centre of the table.



Did You Know?

A *carpenter's square* is used in construction to draw and confirm right angles.

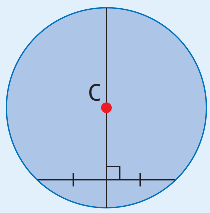


Show You Know

Mark would like to plant a cherry tree in the centre of a circular flower bed. Explain how he could identify the exact centre using circle properties.

Key Ideas

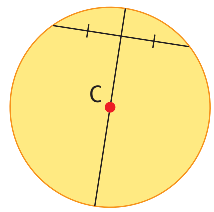
- The perpendicular bisector of a chord passes through the centre of the circle.
- The perpendicular bisectors of two distinct chords intersect at the centre of the circle.
- If a bisector of a chord in a circle passes through the centre, then the bisector is perpendicular to the chord.
- If a line passes through the centre of a circle and intersects a chord at right angles, then the line bisects the chord.
- The shortest path between the centre of a circle and a chord is a line that is perpendicular to the chord.



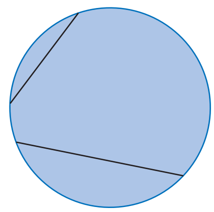
Check Your Understanding

Communicate the Ideas

1. Describe how you know that the diameter of the circle forms a right angle with the chord at their point of intersection.



2. Explain how you could locate the centre of the circle using the two chords shown.



3. Amonte was explaining the properties of perpendicular bisectors to his friend Darius.

"There are three important properties of perpendicular bisectors of chords in circles:
- The line bisector cuts the chord in two equal line segments.
- The line intersects the chord at right angles; they are perpendicular.
- The line passes through the centre so it contains the diameter.

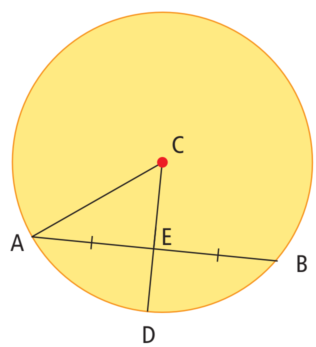
If any two of these properties are present, then the third property exists."

Is Amonte's explanation correct? What does he mean by the last statement?

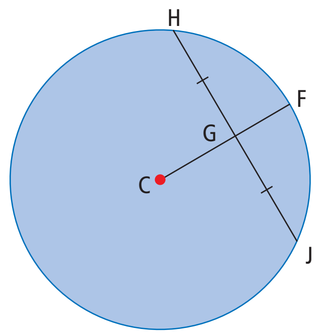
Practise

For help with #4 and #5, refer to Example 1 on page 387.

4. CD bisects chord AB. The radius of the circle is 15 cm long. Chord AB measures 24 cm. What is the length of CE? Explain your reasoning.



5. The radius CF bisects chord HJ. CG measures 4 mm. Chord HJ measures 14 mm. What is the radius of the circle, expressed to the nearest tenth of a millimetre? Justify your answer.

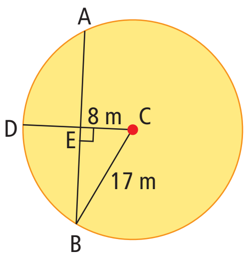


For help with #6, refer to Example 2 on page 388.

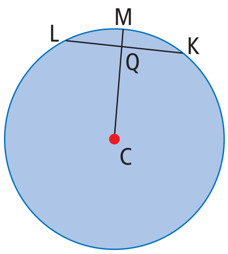
6. Hannah wants to draw a circular target on her trampoline. Explain, using diagrams, how she could locate the centre of the trampoline.

Apply

7. The radius of the circle is 17 m. The radius CD is perpendicular to the chord AB. Their point of intersection, E, is 8 m from the centre C. What is the length of the chord AB? Explain your reasoning.

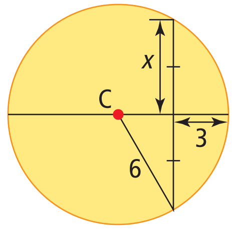


8. The radius of the circle is 11.1 cm, the radius CM is perpendicular to the chord LK, and MQ measures 3.4 cm. What is the length of the chord LK? Express your answer to the nearest tenth of a centimetre.

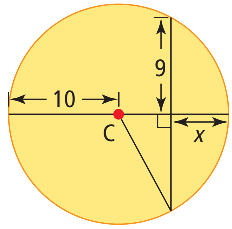


9. Calculate the unknown length, x. Give each answer to the nearest tenth.

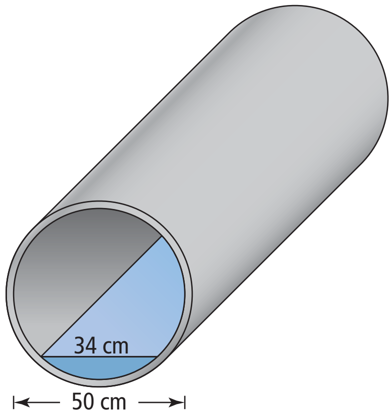
a)



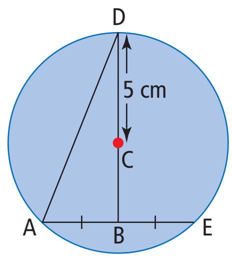
b)



10. The circular cross section of a water pipe contains some water in the bottom. The horizontal distance across the surface of the water is 34 cm. The inner diameter of the pipe is 50 cm. What is the maximum depth of the water? Express your answer to the nearest centimetre.



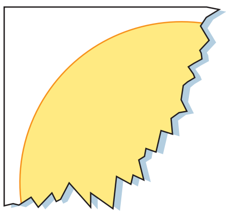
11. If you know that the radius CD = 5 cm, and BC = 3 cm, what is the area of △ABD?



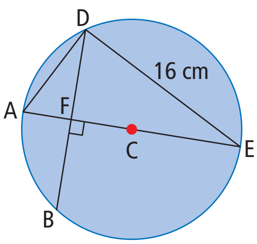
12. A circle has a diameter of 50 mm. A chord is 14 mm long. What is the shortest distance from the centre of the circle, C, to the chord? Include a diagram with your solution.

13. How could you locate the centre of a regular octagonal table using chord properties? Include a diagram in your explanation.

14. Your classmate used a compass to draw a circle with a radius of 8 cm. He felt the circle was inaccurate and tore it into small pieces. How could you use the following piece to check his accuracy?



15. In this circle, the diameter AE = 20 cm, the chord DE = 16 cm, AF = 5 cm, and ∠BFE = 90°.

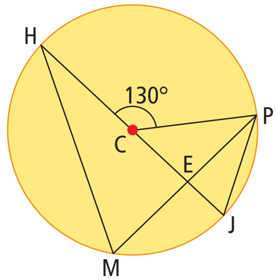


Determine the following measures and justify your answers. Express lengths to the nearest tenth of a centimetre.

a) ∠ADE **b)** AD

c) DF **d)** BD

16. Point E is the midpoint of the chord MP. HJ is a diameter of the circle. C is the centre of the circle, and ∠HCP = 130°.



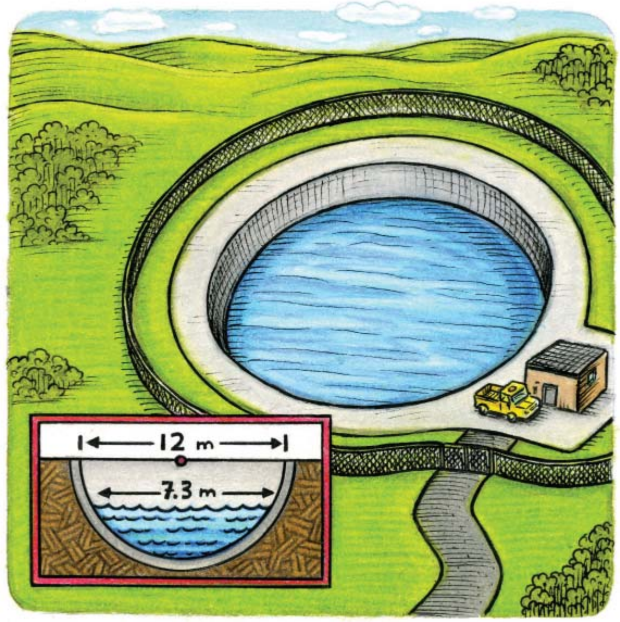
Determine the following angle measures. Justify your answers.

a) ∠HMP **b)** ∠HEM

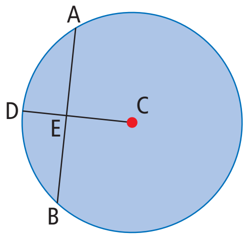
c) ∠MHJ **d)** ∠MPJ

e) ∠PCE **f)** ∠CPE

17. A helicopter pilot surveys the water level in an aqueduct in a remote section of the country. From the air, the pilot measures the horizontal width of the water to be 7.3 m. The aqueduct is a hemisphere and has an inner diameter of 12 m. What is the depth of the water? Express your answer to the nearest tenth of a metre.

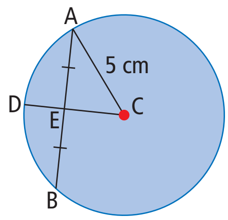


18. Gavyn was asked to find the length of the chord AB. He was told that the radius of the circle is 13 cm, radius CD is perpendicular to chord AB, and chord AB is 5 cm from the centre C.



Determine the mistakes that Gavyn made and find the correct length of AB.

Gavyn's Solution



Draw the radius AC, which is the hypotenuse of right triangle △AEC.

By the Pythagorean relationship,

$$EC^2 + AC^2 = AE^2$$
$$13^2 + 5^2 = AE^2$$
$$169 + 25 = AE^2$$
$$194 = AE^2$$
$$AE = \sqrt{194}$$
$$AE \approx 13.9$$

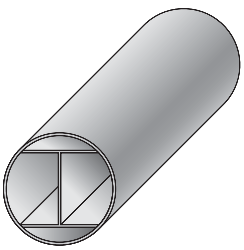
Since CD is a radius and it is perpendicular to AB, then CD bisects the chord AB.

$AB \approx 2 \times 13.9$

$AB \approx 27.8$

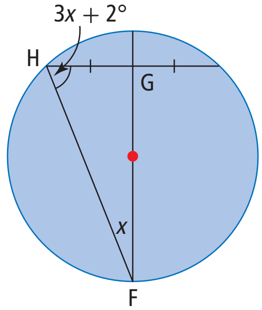
Therefore, AB is approximately 27.8 m.

19. Some plastic tubing is moulded with an I-beam on the inside to provide extra strength. The length of each of two parallel chords is 10 mm, and the perpendicular distance between these two chords is 12 mm. What is the diameter of the circular tubing? Express your answer to the nearest tenth of a millimetre.



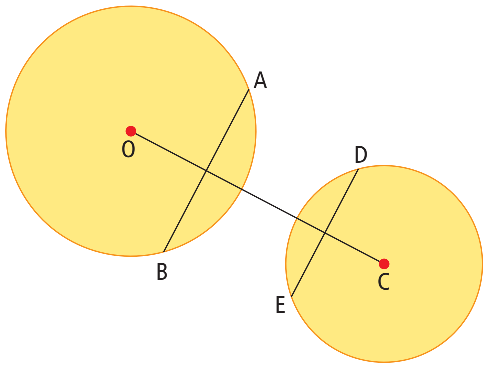
Extend

20. a) How do you know that △FGH is a right triangle?



b) Solve for x algebraically and determine the measures of both acute angles in △FGH.

21. Line segment OC is a bisector of chords AB and DE. If O is the centre of the circle on the left and C is the centre of the circle on the right, explain how you know that AB is parallel to DE.



Math Link

The North American Plains Indians and Tibetan Buddhists create mandalas. A mandala is a piece of art framed within a circle. The design draws the viewer's eyes to the centre of the circle. Mandalas have spiritual significance for their creators. The photo shows a Buddhist monk using coloured sand to create a mandala.

a) Refer to the portion of the sand mandala shown in the picture. Design a mandala with a similar pattern but your own design. For example, you could create a mandala to celebrate the work of a famous mathematician. Your design should show only part of the mandala.

b) If you want to display your mandala, you will need to know how much room the entire design will take up. What is a reasonable estimate for the circumference of your mandala? Explain your reasoning.

c) How do you think the monks ensure symmetry in their mandalas? How could you use your knowledge of circle properties to help you?

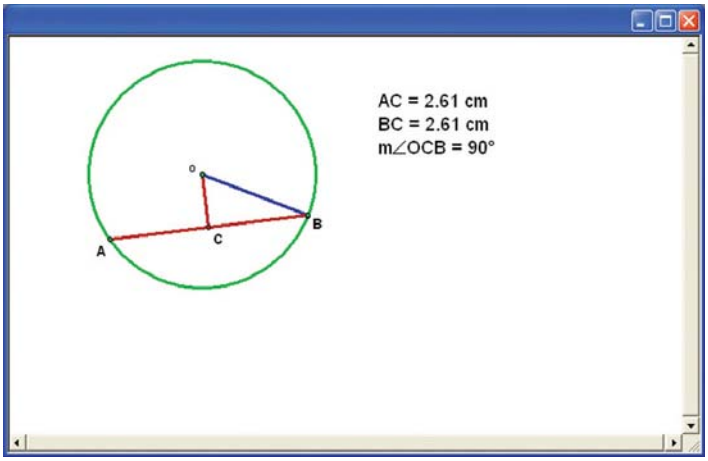
WWW Web Link

For more information about sand mandalas, go to www.mathlinks9.ca and follow the links.

Tech Link

Perpendicular Lines to a Chord

In this activity, you will use dynamic geometry software to explore perpendicular lines from the centre of a circle to a chord. To use this activity, go to www.mathlinks9.ca and follow the links.



Explore

1. a) What is the measure of ∠OCB?

b) What is the measure of line segment AC?

c) What is the measure of line segment BC?

2. Drag point A to another location on the circle.

a) Describe what happens to the measure of ∠OCB when you drag point A to a different location on the circle.

b) What happens to the measures of the line segments AC and BC? Explain.

3. Drag point B around the circle.

a) What effect does this have on the measure of ∠OCB?

b) What effect does this have on the lengths of line segment AC and line segment BC?

4. What conclusions can you make about ∠OCB, the angle formed by the segment from the centre of the circle to the midpoint of the chord?

5. What conclusions can you make about the relationship between line segment AC and line segment BC?