Exploring Angles in a Circle

Focus on...

After this lesson, you will be able to…

- **describe a relationship between inscribed angles in a circle**
- **relate the inscribed angle and central angle subtended by the same arc**

Maddy, Jennifer, and Nia are each about to shoot at the empty net. If they are each equally accurate with their shot, who do you think is most likely to score?

Materials
- compass or circular geoboard with elastic bands
- protractor
- ruler

chord
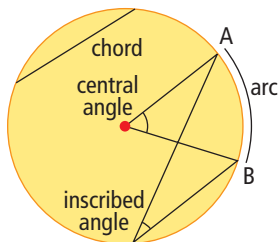- a line segment with both endpoints on a circle

central angle
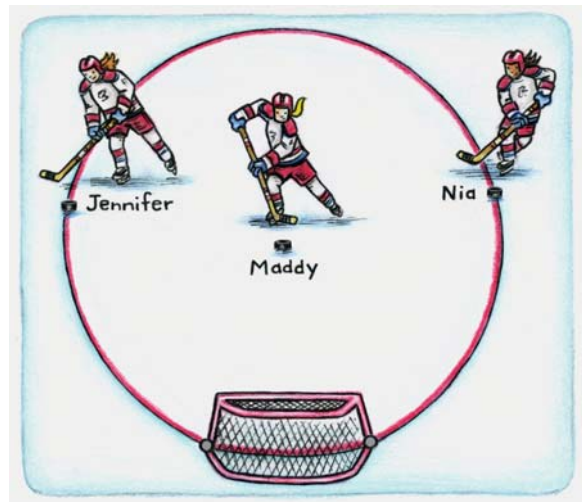- an angle formed by two radii of a circle

inscribed angle
- an angle formed by two chords that share a common endpoint

arc (of a circle)
- a portion of the circumference

Explore Relationships Between Angles in a Circle

1. Construct a large circle and label its centre C. Construct a **chord** AB and a **central angle** ∠BCA. Measure ∠BCA.

2. Create the **inscribed angle** ∠BDA. What is the measure of ∠BDA?

3. How do the measures of ∠BCA and ∠BDA compare?

4. Create a second inscribed angle ∠BEA. What is the measure of ∠BEA?

5. Choose another point on the circle between D and E. Create one more inscribed angle that has its arms touching the endpoints of the **arc** AB. What is the measure of this inscribed angle?

6. Repeat steps 1 to 5 for a different sized circle, and a different sized chord AB.
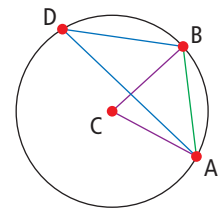
Reflect and Check

7. **a)** What is the relationship between a central angle and an inscribed angle that stands on the same arc?
   b) What is the relationship between all the inscribed angles that stand on the same arc?

8. Predict which hockey player in the opening paragraph is most likely to score on the empty net. Explain.

Link the Ideas

You can use properties related to angles in a circle to solve problems.

Inscribed Angles
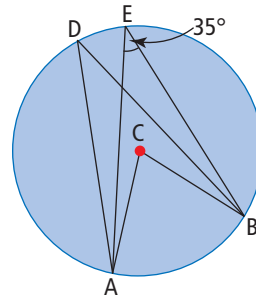The inscribed angles *subtended* by the same arc are congruent.

Central and Inscribed Angles
The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc.

Example 1: Determine Angle Measures in a Circle

Point C is the centre of the circle. $\angle AEB = 35°$

a) What is the measure of $\angle ADB$?
Justify your answer.

b) What is the measure of $\angle ACB$?
Justify your answer.

Solution

a) The inscribed angles, $\angle ADB$ and $\angle AEB$, are equal because they are subtended by the same arc, AB.

Therefore, $\angle ADB = 35°$.

b) The central angle $\angle ACB$ is subtended by the same arc AB as the inscribed angle $\angle AEB$. A central angle is twice the measure of an inscribed angle that is subtended by the same arc.
$$\angle ACB = 2\angle AEB$$
$$= 2 \times 35°$$
$$= 70°$$

Therefore, $\angle ACB = 70°$.

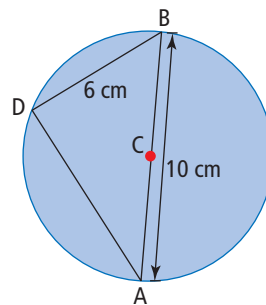**Show You Know**

Point C is the centre of the circle. $\angle DAB = 55°$.
What are the measures of angles $\angle DEB$ and $\angle DCB$?
Justify your answers.
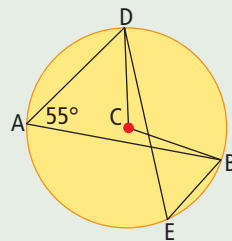
Example 2: Use Central and Inscribed Angles to Recognize Relationships

Point C is the centre of the circle.
diameter AB = 10 cm
chord BD = 6 cm

a) What is the measure of ∠ADB?
   Explain your reasoning.
b) What is the length of the chord AD?
   Justify your answer.

Solution

a) The diameter AB divides the circle into two semicircles. Since AB is a straight line, the central angle ∠ACB is 180°. Then, ∠ADB must be half of 180° because it is an inscribed angle that is subtended by the same arc, AB. The measure of ∠ADB is 90°.

b) Since ∠ADB = 90°, △ABD is a right triangle.
The Pythagorean relationship can be used to find the length of AD.
$$AD^2 + BD^2 = AB^2$$
$$AD^2 + 6^2 = 10^2$$
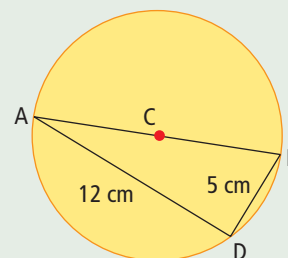$$AD^2 + 36 = 100$$
$$AD^2 = 64$$
$$AD = \sqrt{64}$$
$$AD = 8$$

Therefore, AD = 8 cm.

Show You Know

Point C is the centre of the circle. AB is the diameter.
chord AD = 12 cm
chord BD = 5 cm
a) What is the measure of ∠ADB? Explain your reasoning.
b) What is the length of the diameter AB?

Example 3: Use Central and Inscribed Angles to Solve Problems

Jamie works for a realtor. One of his jobs is to photograph houses that are for sale. He photographed a house two months ago using a camera lens that has a 70° field of view. He has returned to the house to update the photo, but he has forgotten his original lens. Today he only has a telephoto lens with a 35° field of view.
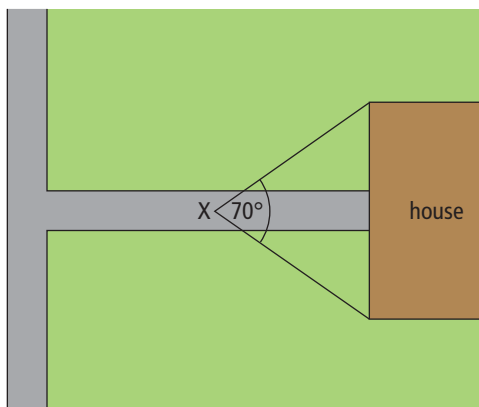
From what location(s) could Jamie photograph the house, with the telephoto lens, so that the entire house still fills the width of the picture. Explain your choices.

Solution

Draw a circle with the centre located at the vertex of the 70° angle. Use one arm of the angle as the radius of the circle. Construct any number of different inscribed angles that each contains the front of the house. Any of these points are locations from which Jamie could take the photo. The measure of each of these inscribed angles will be half the measure of the central angle.

$70° ÷ 2 = 35°$

Each inscribed angle will measure 35°, which corresponds to the field of view for Jamie's telephoto lens. Depending on access, and whether there are any trees or a garden in the way, any point on the major arc that is outside of the house will work.
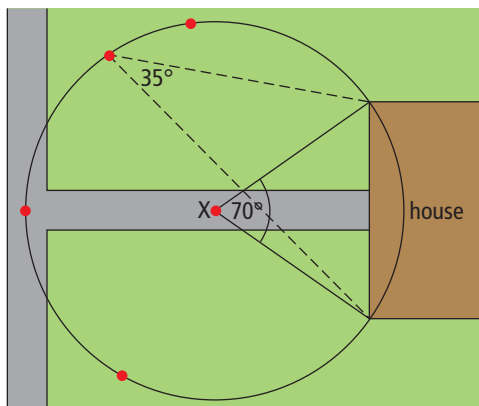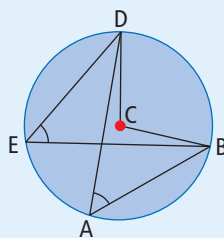
Literacy Link

A *major arc* is more than a semicircle. A *minor arc* is less than a semicircle.

Show You Know

A flashlight has a field of view measuring 25°, and a camera has a field of view measuring 50°. How can you position the camera and flashlight so that the camera will capture the same area as the flashlight illuminates?

Key Ideas

- Inscribed angles subtended by the same arc of a circle are equal. $\angle DEB = \angle DAB$
- A central angle is twice the measure of an inscribed angle subtended by the same arc. $\angle DCB = 2\angle DAB$
- An inscribed angle is one half the measure of a central angle subtended by the same arc. $\angle DAB = \frac{1}{2}\angle DCB$
- When the inscribed angle is subtended by a diameter of the circle, the inscribed angle is equal to 90°.
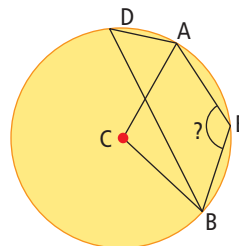

Literacy Link

The identical markings at the inscribed angles $\angle DEB$ and $\angle DAB$ indicate that the measures of these angles are equal.
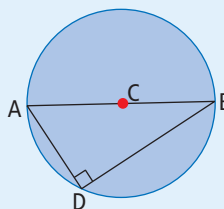
Check Your Understanding

Communicate the Ideas

1. In the diagram, $\angle BDA$ measures half of $\angle BCA$. Does the rule for inscribed angles hold true for $\angle BEA$? Explain your reasoning.
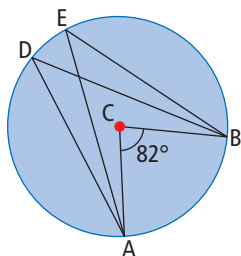

2. Manny constructed a circle using a compass. He used a straight edge to draw a diameter. Then, he constructed an inscribed angle that shared endpoints with the diameter. What is the measure of the inscribed angle he constructed? How do you know?
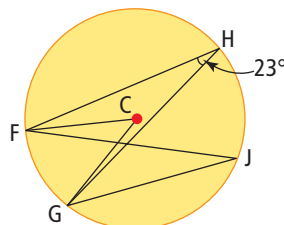
Practise

For help with #3 to #5, refer to Example 1 on page 379.

3. What are the measures of $\angle ADB$ and $\angle AEB$? Justify your answers.
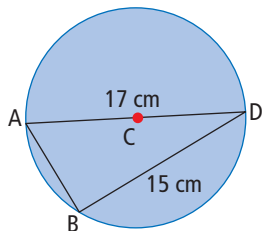

4. a) What is the measure of $\angle FJG$? Explain your reasoning.

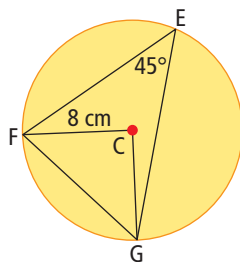**b)** What is the measure of $\angle FCG$? Justify your answer.

5. Draw a circle with a central angle that measures 60°. Draw and label the measure of two inscribed angles that are subtended by the same arc as the central angle.

For help with #6 and #7, refer to Example 2 on page 380.

6. Point C is the centre of the circle.
diameter AD = 17 cm
chord BD = 15 cm

17 cm
A
C
D
15 cm
B

a) What is the measure of ∠ABD? Explain.

b) What is the length of the chord AB?

7. The circle has centre C and a radius of 8 cm.
∠FEG = 45°.

E
45°
8 cm
F
C
G

a) What is the measure of ∠FCG?

b) What is the length of the chord FG? Express your answer to the nearest tenth of a centimetre.

*For help with #8 and #9, refer to Example 3 on page 381.*

8. After a power outage, Jacob helps his mother by shining a flashlight beam at the breaker panel while she locates the tripped breakers. His flashlight projects light through an angle of 15°, while his mother's flashlight projects light through an angle of 30°. Use a diagram to show a good place for Jacob to stand so that his flashlight will illuminate the same area of the breaker panel as his mother's flashlight does.

9. For a high school drama production, three spotlights are positioned on an arc at the back of the theatre, just above the audience. Each spotlight projects light through an angle of 22° and fills the rectangular front of the stage. Use a diagram to identify an ideal location to take a photo of the performance using a camera with a lens that has a field of view of 44°.
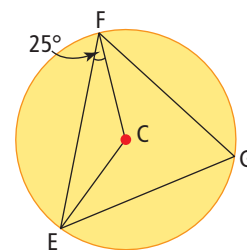

Apply

10. In the diagram, C is the centre of the circle and ∠ABD = 38°. For each of the following questions, justify your answer.

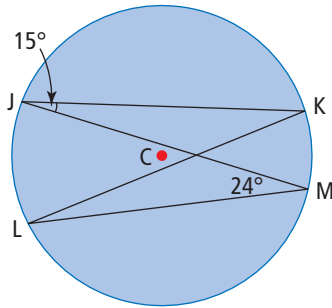38° B
C
A
D
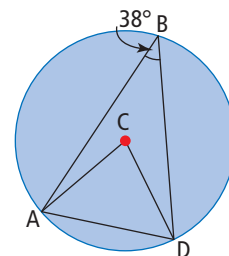
a) What is the measure of ∠ACD?

b) What type of triangle is △ACD?

c) What is the measure of ∠CAD?

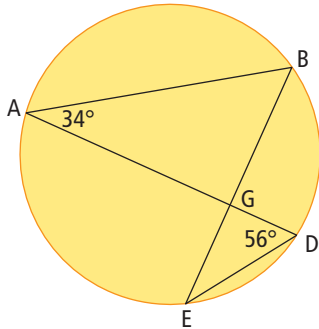**11.** Point C is the centre of the circle and ∠CFE = 25°. Justify each of your answers to the following questions.

F
25°
C
G
E

a) What is the measure of ∠ECF?

b) What is the measure of ∠EGF?

12. If ∠KJM = 15°, ∠JML = 24°, and point C is at the centre of the circle, what is the measure of each of the following angles?


a) ∠KLM   b) ∠JKL

c) ∠JCL   d) ∠KCM

13. In the diagram, ∠BAD = 34° and ∠ADE = 56°.

a) What is the measure of ∠ABE?

b) What is the measure of ∠AGB?

c) What type of triangle is △ABG?

d) What is the measure of ∠DGE?

14. After looking at the diagram of the circle, Amanda decides to use the Pythagorean relationship to calculate the length of chord AB. Will this method work? Explain.
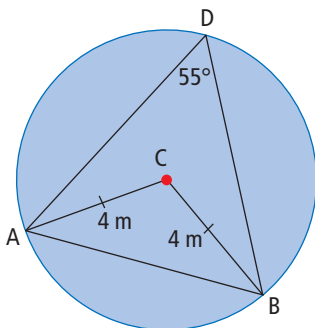

15. Find the unknown angle measures, *x* and *y*, in each diagram. Where C is labelled, it is the centre of the circle.

a)

b)

c)

d)
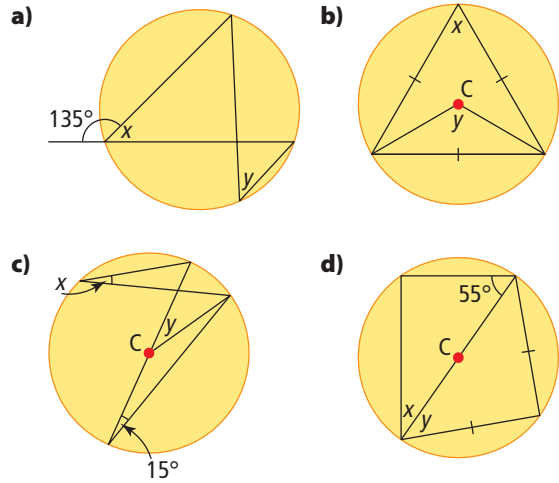
16. Design a geometry question involving a given central angle for which the answer is an inscribed angle measuring 30°. Include a diagram with your question.

17. A circle with centre C has a diameter AB. The inscribed angle ∠ADE measures 14°. What are the measures of ∠ACE and ∠ABE? Draw a diagram.

Extend

18. Find the unknown angle measures, *x* and *y*, in each diagram, given that C is the centre of the circle.
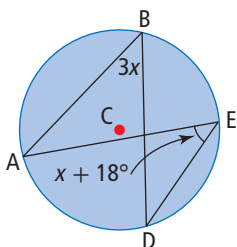
a)

b)
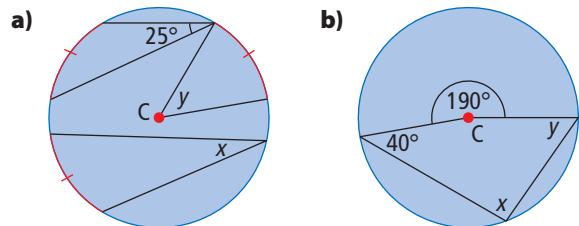
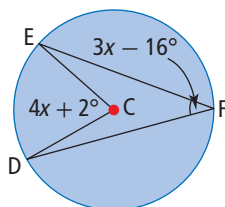**19.** A hole has a diameter of 20 cm. What is the maximum side length of a square that will fit into the hole?

20 cm
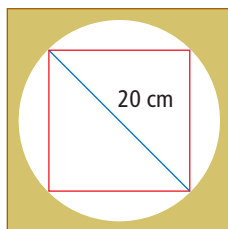
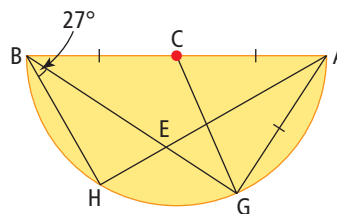**20.** For each of the following diagrams, calculate the value of x.

a)

b)

21. In the semicircle, $\angle HBE = 27°$. C is on the diameter and is the midpoint of AB.

27°

Determine the measure of each angle, justifying your work mathematically.

a) $\angle BHA$

b) $\angle BEH$

c) $\angle AEG$

d) $\angle ACG$

e) $\angle BCG$

Math Link

a) Design a piece of art using one circle and any number of inscribed and central angles.

b) Describe how the angles and line segments in your design are related.
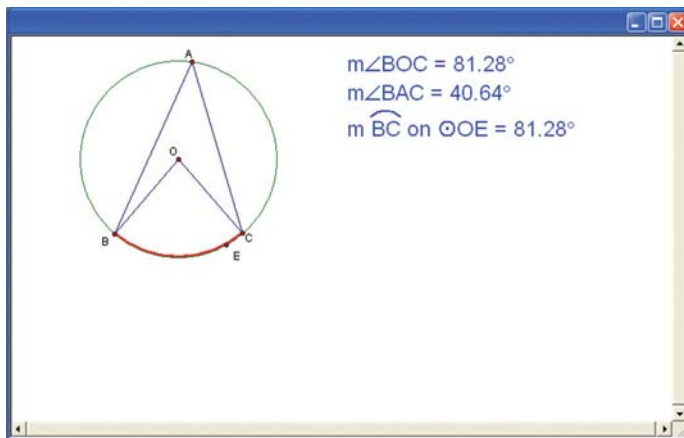
Tech Link

Inscribed and Central Angles

In this activity, you will use dynamic geometry software to explore inscribed and central angles in a circle. To use this activity, go to www.mathlinks9.ca and follow the links.

Explore

1. a) What is the measure of the central angle?

b) What is the measure of the inscribed angle?

c) What is the measure of the minor arc BC?

2. Drag point A around the circle. What happens to the measure of the two angles ∠BOC and ∠BAC? Why does this happen?

3. Drag either point B or point C around the circle. Record at least four measurements of the inscribed angle and the central angle from different locations on the circle.

4. Describe any relationships between the central angle ∠BOC and the inscribed angle ∠BAC subtended by the same arc.

m∠BOC = 81.28°
m∠BAC = 40.64°
m BC on ⊙OE = 81.28°

∠BAC	∠BOC