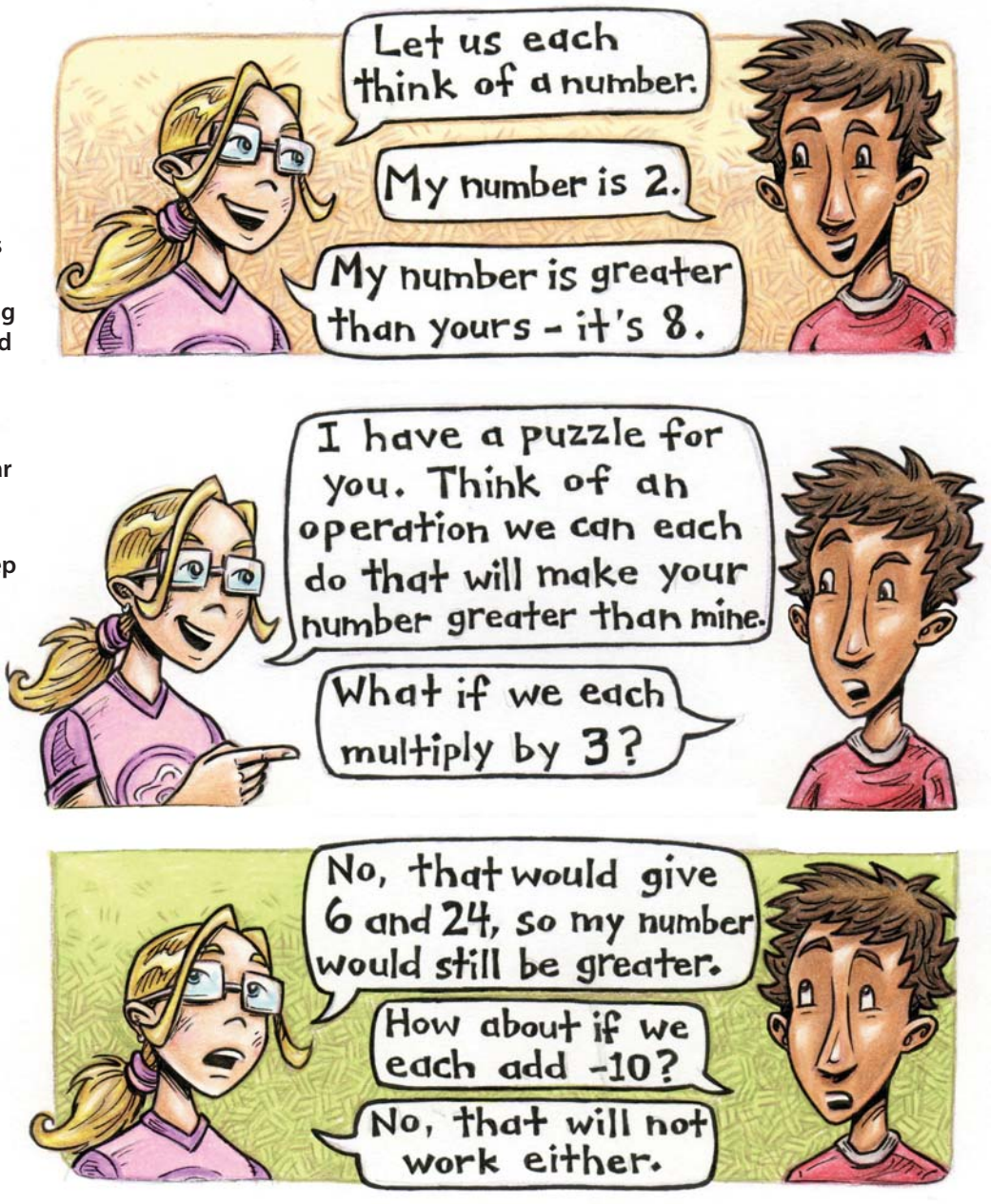
9.2 Solving Single-Step Inequalities

Focus on...

After this lesson, you will be able to...

- solve single-step linear inequalities and verify solutions
- compare the processes for solving linear equations and linear inequalities
- compare the solutions of linear equations and linear inequalities
- solve problems involving single-step linear inequalities



How might you solve Katie's puzzle?

Consider the mathematical operations of addition, subtraction, multiplication, and division. What operations ($+$, $-$, $\times$, $\div$), if any, will reverse the situation so that Joe has the greater number?

Explore Mathematical Operations and Linear Inequalities

Materials
- long strip of paper or number line
- ruler
- two different-coloured tokens or markers

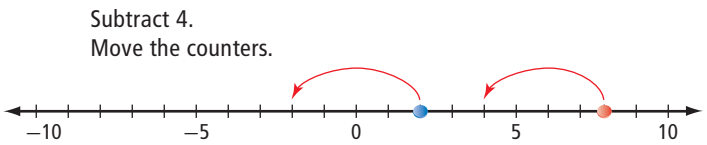
1. On a long strip of paper, draw a number line that shows integers from −20 to 20.

2. a) Work with a partner. Each partner needs to choose an even, positive whole number that is less than 10. Do not choose the same number. Use a different token to show the position of each partner's starting number on the number line.

b) Record an inequality that compares the starting numbers. Note the direction of the inequality symbol and who has the greater number.

c) Choose the same mathematical operation to perform on each partner's number. Move the markers to show the resulting numbers. If necessary, extend your number line.

Whose resulting number is greater? Record an inequality that compares these numbers.



d) Starting each time with your original numbers and inequality, take turns to choose a different mathematical operation and perform it. Each time, move the counters. Whose number is greater? Record the resulting inequality.

e) Try different operations until you are able to predict which operations will reverse an inequality symbol and which ones will keep it the same. Organize your observations and results.

Strategies
Identify All Possibilities

3. a) Conduct a new trial by choosing one negative and one positive number. Use these starting numbers to test your predictions in #2e).

b) Model each operation using the number line and markers. Record your results.

Reflect and Check

4. Consider how the markers moved on the number line.

a) What mathematical operations changed the direction of the inequality symbol? Explain.

b) What operations kept the inequality symbol the same? Explain.

c) Develop an example to support your explanation for parts a) and b).

5. Review your strategy for solving Katie's puzzle. What advice would you give about an operation that would make Joe's number greater than Katie's?

Link the Ideas

Example 1: Solve Inequalities

Solve each inequality.

a) $-2x < 8$ **b)** $x - 3 \geq 2$ **c)** $-5 > \frac{x}{3}$

Solution

a) *Method 1: Use a Model*

You can model the inequality $-2x < 8$ using blocks.

$-2x < 8$

The left side models the *less than* side of the inequality.

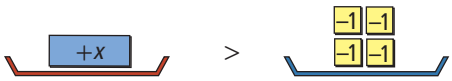
How can you separate the blocks on both sides of the model into two equal groups?

The model shows the inequality with two negative x-blocks on the left side and eight positive unit blocks on the right side. In order for the left side to be less than the right side, each negative x-block must be less than four positive unit blocks.

$-x < 4$

What values of x will make this inequality true?

The inequality $-x < 4$ will be true for $x > -4$. Notice that each side has changed its sign and the inequality symbol is reversed. Represent this solution using blocks. The side with the positive x-block is now greater than the right side.



$x > -4$

Why is the following true?

$-x < 4$

$= -x + x - 4 < 4 + x - 4$

$= -4 < x$

The solution to the inequality $-2x < 8$ is $x > -4$.

Method 2: Isolate the Variable

$$-2x < 8$$

$$\frac{-2x}{-2} > \frac{8}{-2}$$

$$x > -4$$

When dividing by a negative number on both sides, why do you reverse the inequality symbol?

The solution to the inequality is $x > -4$.

b) Isolate the variable.

$$x - 3 \geq 2$$
$$x - 3 + 3 \geq 2 + 3$$
$$x \geq 5$$

When adding a positive number to both sides, what happens to the inequality symbol?

The solution to the inequality is $x \geq 5$.

c) Isolate the variable.

$$-5 > \frac{x}{3}$$
$$(-5) \times 3 > \frac{x}{3} \times 3$$
$$-15 > x$$
$$x < -15$$

How does multiplying by a positive number on both sides affect the inequality symbol?

The solution to the inequality is $x < -15$.

Show You Know

Solve each inequality.

a) $x - 1.6 \leq -5.6$ **b)** $-10 > 4x$ **c)** $\frac{x}{-8} > 3$

Example 2: Verify Solutions to Inequalities

Trevor was asked to solve the inequality $-2x \geq 11$. He represented his solution, $x \geq -5.5$, on a number line. Verify whether Trevor's **solution of the inequality** is correct.

solution of an inequality

- a value or set of values that satisfies an inequality
- can contain many values

What values might you use to verify the solution?

Solution

Substitute some possible values of x into the original inequality:
- Check that the value of the boundary point is correct.
- Check that the inequality symbol is correct.

If the number line is correct, the boundary point of -5.5 should make the two sides of the inequality the same.

Strategies
Guess and Check

Substitute -5.5 into the inequality.

Check:

$$-2x = 11$$
$$-2(-5.5) = 11$$
$$11 = 11$$

True statement

The two sides are equal.
Therefore, -5.5 is the correct boundary point.

If the number line is correct, any value greater than -5.5 should make a true statement.

Substitute one or more values greater than -5.5, such as -5 and 0, into the inequality.

Choose numbers that are easy to work with.

Check:

$-2x \geq 11$	$-2x \geq 11$
$-2(-5) \geq 11$	$-2(0) \geq 11$
$10 \geq 11$	$0 \geq 11$
False statement	False statement

The values -5 and 0 are non-solutions since they result in false statements.

Substituting numbers greater than -5.5 does not result in true statements.

Trevor has drawn the arrow facing the wrong way on the number line. He should have changed the direction of the inequality symbol in his solution. The solution should be $x \leq -5.5$.

Verify the correct solution by substituting one or more values less than -5.5, such as -8 and -6, into the inequality.

Check:

$-2x \geq 11$	$-2x \geq 11$
$-2(-8) \geq 11$	$-2(-6) \geq 11$
$16 \geq 11$	$12 \geq 11$
True statement	True statement

The values -8 and -6 are specific solutions since they result in true statements.

Trevor's solution is not correct.
He forgot to reverse the inequality sign when dividing by a negative number.

Show You Know

Verify the solution for each inequality.
If incorrect, what is the solution?

a) For the inequality $x - 12 \leq 20$, the solution is $x \leq 32$.

b) For the inequality $-5x < 30$, the solution is $x < -6$.

Example 3: Model and Solve a Problem

A games store is offering games on sale for $12.50, including tax. Sean has set his spending limit at $80. How many games can Sean buy and stay within his limit?

a) Write an inequality to model the problem.
b) Solve the inequality and interpret the solution.

Solution

a) If n represents the number of games that Sean can buy, the cost of n games is 12.5 times n. Sean must spend no more than $80.

The situation can be modelled with the inequality $12.5n \leq 80$.

b) $12.5n \leq 80$

$$\frac{12.5n}{12.5} \leq \frac{80}{12.5}$$

$$n \leq 6.4$$

Sean can buy up to and including six games and stay within his spending limit.

$12 \times 6 = 72$ $12 \times 7 = 84$

The number of games Sean can buy is between 6 and 7.

Since it is not possible to buy part of a game, 6.4 is not a solution to the original problem, even though it is a solution to the inequality $12.5n \leq 80$. Only a whole number is a possible solution for this situation.

Show You Know

Yvonne is planting trees as a summer job. She gets paid $0.10 per tree planted. She wants to earn at least $20/h. How many trees must she plant per hour in order to achieve her goal?

a) Write an inequality to model the number of trees Yvonne must plant to reach her goal.
b) Will the solution be a set of whole numbers or a set of integers? Explain.
c) Solve the inequality and interpret the solution.

Did You Know?

Piecework is work paid by the amount done, not by the time it takes. For example, tree planters are paid by the number of trees they plant.

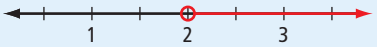
Key Ideas

- The solution to an inequality is the value or values that makes the inequality true.

  $5x > 10$

  A specific solution is any value greater than 2. For example, 2.1, 3, or 22.84.

  The set of all solutions is $x > 2$.



- You can solve an inequality involving addition, subtraction, multiplication, and division by isolating the variable.

$x - 3 \leq 5$	$8x \leq 24$	$\frac{x}{-2} > 6$
$x - 3 + 3 \leq 5 + 3$	$\frac{8x}{8} \leq \frac{24}{8}$	$\frac{x}{-2} \times -2 < 6 \times -2$
$x \leq 8$	$x \leq 3$	$x < -12$

Reverse the inequality symbol when multiplying or dividing both sides by a negative number.

- To verify the solution to an inequality, substitute possible values into the inequality:
  - Substitute the value of the boundary point to check if both sides are equal.
  - Substitute specific value(s) from the solution to check that the inequality symbol is correct.

Check if $x \geq -3$ is the solution to $-8x \leq 24$.

Substitute the boundary point -3.

$$-8x = 24$$
$$-8(-3) = 24$$
$$24 = 24$$

The two sides are equal. Therefore, -3 is the correct boundary point.

Substitute a value greater than the boundary point -3.

$$-8x \leq 24$$
$$-8(0) \leq 24$$
$$0 \leq 24$$

Substituting a value greater than -3 results in a true statement. Therefore, the inequality symbol is correct.

Check Your Understanding

Communicate the Ideas

1. Maria and Ryan are discussing the inequality $2x > 10$.

Maria: The solution to the inequality is 6. When I substitute 6 for x, a true statement results.

Ryan: I agree that 6 is a solution but it is not the whole solution.

What does Ryan mean?

2. Explain how the process for verifying a solution is different for a linear inequality than for a linear equation. Discuss your answer with a classmate.

3. What process would you use to solve the inequality $-15x \leq 90$?

4. Represent on a number line
- the linear equation $6x = 18$
- the linear inequality $6x \geq 18$

Compare the solutions. How are they the same? How are they different?

Practise

For help with #5 to #8, refer to Example 1 on pages 352–353.

5. Solve each inequality.

a) $x - 7 \geq 22$ **b)** $4 < x + 11$

c) $8.6 + x > -5.2$ **d)** $100 \leq x + 65$

6. Solve each inequality.

a) $6y \geq 54$ **b)** $29 > -2y$

c) $3.1y \leq -12.4$ **d)** $-1.6y < -10$

7. Solve each inequality.

a) $\frac{x}{5} > 30$ **b)** $\frac{x}{-4} \geq -9$

c) $2 \geq \frac{x}{1.2}$ **d)** $-\frac{1}{6}x < 5$

8. Look at the following operations. For each one, does the inequality symbol need to be reversed when the operation is performed on both sides of an inequality? Why or why not?

a) Subtract 5.

b) Multiply by 6.

c) Add -15.

d) Divide by -3.

e) Multiply by -19.7.

f) Divide by 0.3.

For help with #9 to #13, refer to Example 2 on pages 353–354.

9. Verify whether the specific solution is correct for each inequality.

a) $x - 2.5 \leq 10; x = 12$

b) $3x \geq 21; x = 8$

c) $-4x < 20; x = -3$

d) $-\frac{1}{5}x \leq 3; x = -20$

10. Verify whether the specific solution satisfies each inequality.

a) $y - 10.2 \geq 18; y = 30$

b) $-6y \leq 36; y = -7$

c) $\frac{-2}{3}y \geq 10; y = 10$

d) $\frac{1}{2}y < 13; y = -2$

11. Show whether $x < 4$ is the solution for each inequality.

a) $-3x > -12$

b) $10 + x > 14$

c) $1 > \frac{x}{4}$

d) $-x > -4$

12. Verify that the solution shown on each number line is correct.

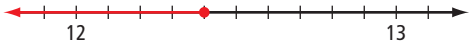
a) $x + 10 > 14$

b) $-3.2 < \frac{x}{5}$

13. Verify each solution represented graphically.

a) $-10 \geq x - 1$

b) $-5x \geq -62$



For help with #14 and #15, refer to Example 3 on page 355.

14. The Super Fencing Company builds cedar fences for homes at a cost of $85 per section of fence, including tax. How many sections of fence could you buy if you could spend no more than $1400?

a) Model the problem using an inequality.

b) Solve the inequality.

c) Is the boundary point a reasonable solution for the number of fence sections? Explain.

15. Megan is competing in a series of mountain bike races this season. She gets 6 points for each race she wins. If she gets more than 50 points in total, she will move up to the next racing category. How many race wins this season will allow her to move up to the next category?

a) Use an inequality to represent the problem.

b) Determine the solution and use it to solve the problem.

c) Is the boundary point a reasonable solution for the number of race wins? Explain.

Apply

16. For each of the following inequalities, state three values that are specific solutions and three values that are non-solutions.

a) $-5 + x < -10$

b) $-3x < 24$

17. Colin's teacher asked him to solve the inequality $-5x \geq -15$. His solution was $x \leq 3$. He explained that he reversed the inequality symbol because of the negative number. Write a more accurate explanation.

18. A local sports complex offers the following options for sharpening skates.

Skate Sharpening Rates

Standard Rate: $5.75 per pair of skates

Special: $49 per month for unlimited sharpening

a) Estimate at what point the special would be the better option. Show the process you used. Why do you think your method provides a reasonable estimate?

b) Model and solve the problem using an inequality. Compare the answer to your estimate.

19. The owner of a craft store donates 3% of her profits to a local charity every month. If she wants to donate at least $250 this month, how much profit will the business need to earn?

a) Model and solve the problem using an inequality.

b) Verify your solution. Show your work.

20. Andrew's family is driving from Winnipeg to Saskatoon. Before leaving, they fill the gas tank with 57 L of fuel. The car uses fuel at an average rate of 8.4 L/100 km for highway driving. How many kilometres can they drive on this amount of fuel? What assumptions did you make?

21. Natalie is entering the 3200-m event at an upcoming meet. Each lap of the track is 400 m. Her goal is to beat the current record of 9 min 23 s. How fast must she run each lap, on average, to beat the record?

a) Explain why the situation can be modelled using the inequality $8x < 563$.

b) Solve the problem and verify your solution. Show your work.

22. Fiona has a rewards card that gives her a reward point for every $5 she spends. If she earns at least 120 points in a year, she gets a bonus. How much does she need to spend to get at least 120 points?

Extend

23. Chris has a weekend business building doghouses. Each doghouse takes 4 h to build and is sold for $115. Chris wants to earn at least $1000 per month. He wants to work no more than 50 h on his business per month.

a) Write two inequalities to model the situation.

b) Solve each inequality.

c) What possible numbers of doghouses can he build each month and stay within his guidelines?

24. Solve and check the inequality $-\frac{2}{5}x < \frac{1}{3}$. Show the solution on a number line.

25. If $-2x > 22$ and $-4x < 60$, determine the possible values of x that satisfy both inequalities. Show your solution on a number line.

26. A food company that is developing a new energy bar has not decided on the size of the bar. The recipe includes 9% protein and 13% fat. The company wants the bar to contain at least 6 g of protein and no more than 10 g of fat. Use two inequalities to determine the possible range of masses for the bar.

27. Consider the inequality $ax \leq 5a$.

a) Solve the inequality if $a > 0$.

b) Solve the inequality if $a < 0$.

28. Solve each combination of inequalities.

a) $-5 \leq x + 9$ and $x + 9 \leq 8$

b) $-2 < 2x$ and $2x < 12$

c) $-15 \leq -6x$ and $-6x < 9$

Math Link

Some amusement parks offer single-ride tickets, where you pay each time you ride, and all-day passes, where you pay once for unlimited rides. The prices for both types of tickets need to be high enough for the amusement park to earn a profit but low enough that people decide to come.

Search various media, such as newspapers, magazines, and the Internet. Look for information about ticket prices at amusement parks.

a) Choose a price for single-ride tickets and a price for all-day passes. Explain why your choices are reasonable.

b) Use an inequality to determine the number of rides that make one option a better deal than the other.

c) Your friends plan on going on seven rides in your amusement park. Which is the better option for them? Show your work.