Solving Equations: $a(x + b) = c$

Each year, Canada's Prairie Provinces produce tens of millions of tonnes of grains, such as wheat, barley, and canola. The growth of a grain crop partly depends on the quantity of heat it receives. One indicator of the quantity of heat that a crop receives in a day is the *daily average temperature*. This is defined as the average of the high and low temperatures in a day.

How can you calculate the daily average temperature on a day when the high temperature is 23 °C and the low temperature is 13 °C? If the low temperature is 10 °C, how could you determine the high temperature that would result in a daily average temperature of 15 °C?

What equations can you use to represent these situations?

Explore Equations With Grouping Symbols

Materials
- coins
- paper cups or small containers
- paper clips

1. Explain how the diagram models the solution to the equation $2(x + 0.10) = 0.30$. What is the solution?
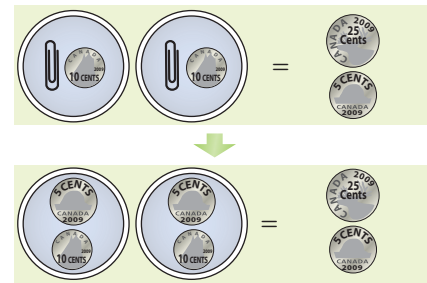
2. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $2(x + 0.25) = 0.54$

b) $3\left(x + \dfrac{1}{20}\right) = \dfrac{9}{10}$

Reflect and Check

3. a) How can you model solutions to equations of the form $a(x + b) = c$ using manipulatives or diagrams?

   b) Think of other ways to model the solutions. Explain how you would use them.

4. Stefan works on a farm in the Fraser Valley of British Columbia. Two of the fields are square. The perimeter of the larger field is 2.4 km. The side length of the larger field is 0.1 km more than the side length of the smaller field. Create a labelled drawing of this situation. Suggest ways of determining the side length of the smaller field. Include a suggestion for solving an equation of the form $a(x + b) = c$. Share your ideas with your classmates.

Link the Ideas

Example 1: Solve Equations With Grouping Symbols

Solve and check.

a) $3(d + 0.4) = -3.9$

b) $\dfrac{t - 1}{5} = \dfrac{3}{2}$

Solution

a) *Method 1: Use the Distributive Property First*
Use the distributive property to remove the brackets.

$$3(d + 0.4) = -3.9$$
$$(3 \times d) + (3 \times 0.4) = -3.9$$
$$3d + 1.2 = -3.9$$
$$3d + 1.2 - 1.2 = -3.9 - 1.2$$
$$3d = -5.1$$
$$\dfrac{3d}{3} = \dfrac{-5.1}{3}$$
$$d = -1.7$$

> **Literacy Link**
>
> The *distributive property* is:
> $a(b + c) = ab + ac$

Method 2: Divide First

$$3(d + 0.4) = -3.9$$
$$\frac{3(d + 0.4)}{3} = \frac{-3.9}{3}$$
$$d + 0.4 = -1.3$$
$$d + 0.4 - 0.4 = -1.3 - 0.4$$
$$d = -1.7$$

Why do you divide both sides by 3?

Check:
Left Side $= 3(d + 0.4)$ Right Side $= -3.9$
$= 3(-1.7 + 0.4)$
$= 3(-1.3)$
$= -3.9$

Left Side = Right Side
The solution, $d = -1.7$, is correct.

b)
$$\frac{t - 1}{5} = \frac{3}{2}$$
$$10 \times \frac{t - 1}{5} = 10 \times \frac{3}{2}$$
$$2(t - 1) = 15$$
$$2t - 2 = 15$$
$$2t - 2 + 2 = 15 + 2$$
$$2t = 17$$
$$\frac{2t}{2} = \frac{17}{2}$$
$$t = \frac{17}{2}$$

Why do you multiply both sides by 10? Is there a different way to solve the equation?

Literacy Link

A *fraction bar* acts as a grouping symbol and as a division symbol.
The expression $\frac{t - 1}{5}$ can be written as $\frac{1}{5}(t - 1)$ or as $(t - 1) \div 5$.

Check:
Left Side $= \frac{t - 1}{5}$ Right Side $= \frac{3}{2}$
$= \left(\frac{17}{2} - 1\right) \div 5$
$= \left(\frac{17}{2} - \frac{2}{2}\right) \div 5$
$= \frac{15}{2} \div \frac{5}{1}$
$= \frac{15}{2} \times \frac{1}{5}$
$= \frac{3}{2}$

Left Side = Right Side
The solution, $t = \frac{17}{2}$, is correct.

Show You Know

Solve and check.

a) $2(e - 0.6) = 4.2$ **b)** $\frac{c + 2}{3} = -\frac{5}{2}$

Example 2: Apply Equations With Grouping Symbols

On a typical February day in Whitehorse, Yukon Territory, the daily average temperature is -13.2 °C. The low temperature is -18.1 °C. What is the high temperature?

Solution

Let the high temperature be T degrees Celsius. The daily average temperature, in degrees Celsius, is the average of the high and low temperatures, or $\dfrac{T + (-18.1)}{2}$. The daily average temperature is -13.2 °C.

> How could you estimate the high temperature?

An equation that represents this situation is $\dfrac{T + (-18.1)}{2} = -13.2$.

Isolate the variable, T.

$$\frac{T + (-18.1)}{2} = -13.2$$

$$2 \times \frac{T + (-18.1)}{2} = 2 \times (-13.2)$$

$$T - 18.1 = -26.4$$

$$T - 18.1 + 18.1 = -26.4 + 18.1$$

$$T = -8.3$$

The high temperature is -8.3 °C.

Check:

The average of the high and low temperatures is $\dfrac{-8.3 + (-18.1)}{2}$.

$$\frac{-8.3 + (-18.1)}{2} = \frac{-26.4}{2}$$

$$= -13.2$$

The calculated average of -13.2 °C agrees with the daily average temperature given in the problem.

> **Show You Know**
>
> On a typical day in October in Churchill, Manitoba, the daily average temperature is -1.5 °C. The high temperature is 1.3 °C. Estimate and then calculate the low temperature.

Barren Lands, Churchill, Manitoba

Key Ideas

- To isolate the variable in an equation of the form $a(x + b) = c$, you can

  ▪ use the distributive property first

  $$4(r - 0.6) = -3.2$$
  $$4r - 2.4 = -3.2$$
  $$4r - 2.4 + 2.4 = -3.2 + 2.4$$
  $$4r = -0.8$$
  $$\frac{4r}{4} = \frac{-0.8}{4}$$
  $$r = -0.2$$

  ▪ divide first

  $$4(r - 0.6) = -3.2$$
  $$\frac{4(r - 0.6)}{4} = \frac{-3.2}{4}$$
  $$r - 0.6 = -0.8$$
  $$r - 0.6 + 0.6 = -0.8 + 0.6$$
  $$r = -0.2$$

- To solve equations involving grouping symbols and fractions, you can rewrite the equation and work with integers instead of performing fraction operations.

  $$\frac{q - 1}{2} = \frac{3}{4}$$
  $$4 \times \frac{q - 1}{2} = 4 \times \frac{3}{4}$$
  $$2(q - 1) = 3$$
  $$2q - 2 = 3$$
  $$2q - 2 + 2 = 3 + 2$$
  $$2q = 5$$
  $$\frac{2q}{2} = \frac{5}{2}$$
  $$q = \frac{5}{2}$$

- You can check solutions by using substitution.

  $$\text{Left Side} = \frac{q - 1}{2} \qquad\qquad \text{Right Side} = \frac{3}{4}$$
  $$= \left(\frac{5}{2} - 1\right) \div 2$$
  $$= \left(\frac{5}{2} - \frac{2}{2}\right) \div 2$$
  $$= \frac{3}{2} \times \frac{1}{2}$$
  $$= \frac{3}{4}$$

  $$\text{Left Side} = \text{Right Side}$$
  $$\text{The solution, } q = \frac{5}{2}, \text{ is correct.}$$

- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

1. Mario solved the equation $2(n + 1.5) = 4.5$ as follows.

$$2(n + 1.5) = 4.5$$
$$2n + 3 = 9$$
$$2n = 6$$
$$n = 3$$

a) What is the error in his reasoning? Explain.

b) Write the correct solution.

2. Cal and Tyana solved the equation $3(k - 4.3) = -2.7$ in different ways. Cal used the distributive property first, while Tyana divided first.

  a) Show both of their methods.

  b) Whose method do you prefer? Explain.

  c) If the equation was $3(k - 4.3) = -2.5$, which method would you use? Explain.

3. Renée and Paul used different methods to solve $\dfrac{x + 1}{2} = \dfrac{3}{5}$.

  a) Renée first multiplied both sides by a common multiple.

$$10 \times \dfrac{x + 1}{2} = 10 \times \dfrac{3}{5}$$

  Show the rest of her solution.

  b) Paul first rewrote the equation as $\dfrac{1}{2}(x + 1) = \dfrac{3}{5}$ and used the distributive property. Show the rest of his solution.

$$\dfrac{1}{2}x + \dfrac{1}{2} = \dfrac{3}{5}$$

  c) Which solution do you prefer? Explain.

**4.** Viktor and Ashni were solving the following problem together.

A square with a side length of $x + 1$ has a perimeter of 18.6 units. What is the value of x?

They disagreed over how to model the situation with an equation.
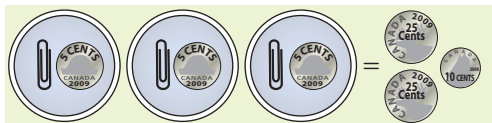
Viktor's Equation   *Ashni's Equation*
$4(x + 1) = 18.6$ $4x + 1 = 18.6$

  a) Which equation is correct? Explain.

  b) What is the value of x?

  c) How can you check whether your value for x is correct?

$x + 1$
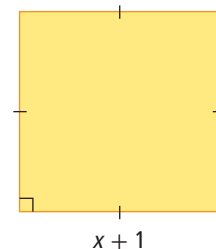
Practise

5. Write an equation that is represented by the following. Then, solve the equation.

For help with #6 to #9, refer to Example 1 on pages 315–316.

6. Solve and check.

  a) $2(x + 1.5) = 7.6$

  b) $-2.8 = -1(c - 0.65)$

  c) $-3.57 = 3(a + 4.51)$

  d) $-3.6(0.25 - r) = 0.18$

7. Solve. Express each solution to the nearest hundredth.

  a) $3(u - 12.5) = -3.41$

  b) $14.01 = -7(1.93 + m)$

  c) $6(0.15 + v) = 10.97$

  d) $-9.5(x - 4.2) = 7.5$

8. Solve and check.

  a) $\dfrac{n + 1}{2} = -\dfrac{3}{4}$ **b)** $\dfrac{5}{2} = \dfrac{1}{3}(x - 2)$

  c) $\dfrac{3}{4}(w + 2) = 1\dfrac{1}{3}$ **d)** $\dfrac{7}{6} = \dfrac{2(5 - g)}{3}$

9. Solve.

a) $\dfrac{1-y}{3} = \dfrac{2}{5}$

b) $-\dfrac{1}{2}(q+4) = 2\dfrac{1}{4}$

c) $-\dfrac{7}{10} = \dfrac{e+3}{5}$

d) $\dfrac{2(p-3)}{3} = \dfrac{1}{2}$

For help with #10 and #11, refer to Example 2 on page 317.

10. Solve and check.

a) $\dfrac{x+4.1}{3} = 2.5$

b) $19.8 = \dfrac{4.2+k}{-3}$

c) $\dfrac{q-6.95}{2} = -4.61$

d) $-2.1 = \dfrac{4.6-a}{-5}$

11. Solve.

a) $-0.25 = \dfrac{q-1.6}{2}$

b) $\dfrac{y+0.385}{-1} = -0.456$

c) $\dfrac{7.34+n}{4} = 1.29$

d) $7.56 = \dfrac{p-15.12}{-2}$

Apply

12. The mean of two numbers is 3.2. One of the numbers is 8.1. What is the other number?

13. Two equilateral triangles differ in their side lengths by 1.05 m. The perimeter of the larger triangle is 9.83 m. Determine the side length of the smaller triangle by

a) representing the situation with an equation of the form $a(x+b) = c$, and solving the equation.

b) using a different method of your choice. Explain your reasoning.

14. The regular pentagon has a perimeter of 18.8 units. What is the value of x?

$x - 3$
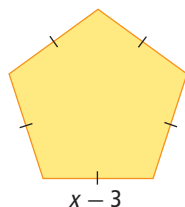
15. On a typical January day in Prince Rupert, British Columbia, the daily average temperature is -0.2 °C. The low temperature is -3.7 °C. What is the high temperature?

16. A regular hexagon has a perimeter of 41.4 units. The side length of the hexagon is represented by the expression $2(3-d)$. What is the value of d?

17. Henri bought three jars of spaghetti sauce. He used a coupon that reduced the cost of each jar by $0.75. If he paid $6.72 altogether, what was the regular price of each jar?

18. Luisa bought five concert tickets. She paid a $4.50 handling fee for each ticket. The total cost, before tax, was $210.00. What was the cost of each ticket, excluding the handling fee?

19. Mary wants to make her family kamiks, which are boots made from seal or caribou skin. She usually pays $80 for each skin, but Lukasie offers her a discount if she buys five skins. If Mary pays $368, how much did Lukasie reduce the price of each skin?

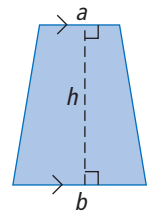**20.** The area of a trapezoid can be found using the formula $A = \dfrac{1}{2}(a+b) \times h$, where a and b are the lengths of the two parallel sides, and h is the distance between them.
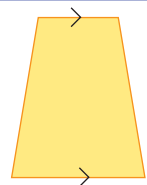

Determine each of the following.

a) h when $A = 27.3$ cm², $a = 2.3$ cm, and $b = 4.7$ cm

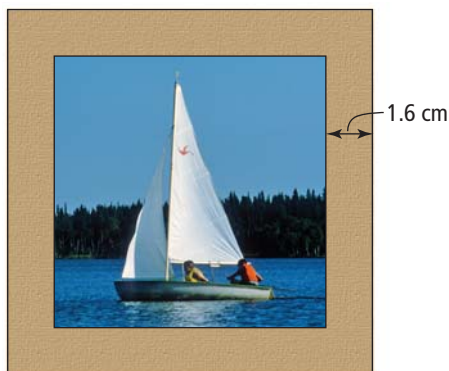b) a when $A = 4.8$ m², $b = 1.9$ m, and $h = 3$ m

> **⊕ Literacy Link**
>
> A *trapezoid* is a quadrilateral with exactly two parallel sides.

21. A square picture frame is made from wood that is 1.6 cm wide. The perimeter of the outside of the frame is 75.2 cm. What is the side length of the largest square picture that the frame will display?


—1.6 cm

22. For a fit and healthy person, the maximum safe heart rate during exercise is approximately related to their age by the formula $r = \frac{4}{5}(220 - a)$. In this formula, r is the maximum safe heart rate in beats per minute, and a is the age in years. At what age is the maximum safe heart rate 164 beats/min?

Extend

23. Solve and check.
   a) $2(x + 3) + 3(x + 2) = 0.5$
   b) $4(y - 3) - 2(y + 1) = -4.2$
   c) $1.5(4 + f) + 2.5(5 - f) = 15.7$
   d) $-5.3 = 6.2(t + 6) - 1.2(t - 2)$

24. Solve.
   a) $4(d + 3) - 3(d - 2) = 1.2$
   b) $-10.5 = 5(1 - r) + 4(r - 3)$
   c) $3.9 = 2.5(g - 4) + 1.5(g + 5)$
   d) $-1.8(h + 3) - 1.3(2 - h) = 1$

25. The area of the trapezoid is 1.5 square units. What are the lengths of the parallel sides?

$3x - 2$

4

$2x - 1$
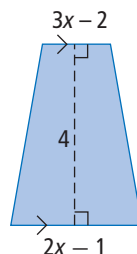
26. If $1.5(x + 1) + 3.5(x + 1) = 7.5$, determine the value of $-10(x + 1)$ without determining the value of x. Explain your reasoning.

27. Tahir is training for an upcoming cross-country meet. He runs 13 km, three times a week. His goal is to increase his average speed by 1.5 km/h, so that he can complete each run in $1\frac{1}{4}$ h. How long does he take to complete each run now, to the nearest tenth of a minute? Solve this problem in two different ways.

28. a) Solve $x(n - 3) = 4$ for n by dividing first.
   b) Solve $x(n - 3) = 4$ for n by using the distributive property first.
   c) Which method do you prefer? Explain.

Math Link

One serving of a breakfast mixture consists of 200 mL of a corn bran cereal and 250 mL of 2% milk. Two servings of the mixture provide 1.4 mg of thiamin. If 250 mL of 2% milk provides 0.1 mg of thiamin, what mass of thiamin is in 200 mL of the cereal?

a) Write an equation that models the situation.

b) Solve the equation in two different ways.

c) Which of your solution methods do you prefer? Explain.

Did You Know?

Thiamin is another name for vitamin B_1. The body needs it to digest carbohydrates completely. A lack of thiamin can cause a loss of appetite, weakness, confusion, and even paralysis. Sources of thiamin include whole grains, liver, and yeast.