

8.2

Solving Equations:

$$ax + b = c, \frac{x}{a} + b = c$$

Focus on...

After this lesson, you will be able to...

- model problems with linear equations involving two operations
- solve linear equations with rational numbers using two operations

WWW Web Link

To learn more about waterfalls in Canada and around the world, go to www.mathlinks9.ca and follow the links.

Two of Canada's highest measured waterfalls are in British Columbia. Takakkaw Falls, is in Yoho National Park, 27 km west of Lake Louise. Its height is 254 m. This is 34 m more than half the height of Della Falls in Strathcona Park on Vancouver Island.

Choose a variable to represent the height of Della Falls. Then, write and solve an equation to find the height of Della Falls.



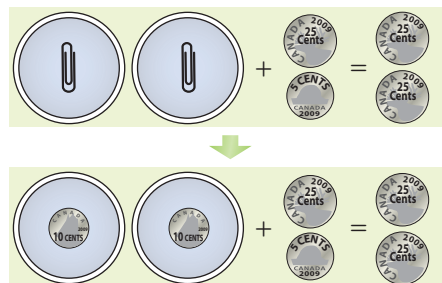
Della Falls, Vancouver Island

Materials

- coins
- cups or small containers
- paper clips

Explore Equations With Two Operations

- How does the diagram model the solution to the equation $2x + 0.30 = 0.50$?
 - What is the solution?



2. a) Explain how the second part of the diagram in #1 can model the equation $0.10y + 0.30 = 0.50$. What is the solution? Explain.
 b) How does the second part of the diagram in #1 also model the solution to the equation $\frac{z}{10} + \frac{3}{10} = \frac{1}{2}$? What is the solution?
3. Describe how you would use manipulatives or diagrams to model the solution to each of the following.
- a) $3x + 0.05 = 0.26$
 b) $0.01x + 0.05 = 0.08$
 c) $\frac{x}{4} + \frac{1}{5} = \frac{7}{10}$
4. Work with a partner to explore how to model the solution to the equation $2x - 0.11 = 0.15$. Share your models with other classmates.

Reflect and Check

5. a) How can you model solutions to equations of the form $ax + b = c$ and $\frac{x}{a} + b = c$ using manipulatives or diagrams?
 b) Think of other ways to model the solutions. Explain how you would use them.
6. The tallest waterfall in the world is Angel Falls in Venezuela, with a height of about 0.8 km. This height is 0.08 km less than twice the height of Della Falls. Write and solve an equation to determine the height of Della Falls in kilometres. Check that your answer agrees with the height in metres you determined at the beginning of this section.



Angel Falls, Venezuela

Did You Know?

Canada's tallest free-standing structure is the CN Tower, with a height of about 550 m. This is about 250 m less than the height of Angel Falls.

Link the Ideas

Example 1: Solve Two-Step Equations With Fractions

Solve and check.

a) $2x + \frac{1}{10} = \frac{3}{5}$

b) $\frac{k}{3} - \frac{1}{2} = -1\frac{3}{4}$

Solution

a) $2x + \frac{1}{10} = \frac{3}{5}$

$$2x + \frac{1}{10} - \frac{1}{10} = \frac{3}{5} - \frac{1}{10}$$

$$2x = \frac{3}{5} - \frac{1}{10}$$

$$2x = \frac{6}{10} - \frac{1}{10}$$

$$2x = \frac{5}{10}$$

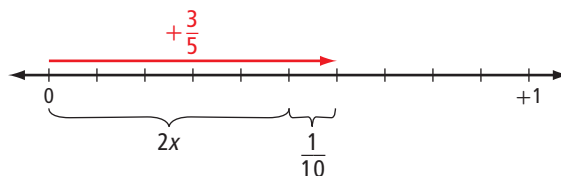
$$2x \div 2 = \frac{1}{2} \div 2$$

$$x = \frac{1}{4}$$

To isolate the variable in a two-step equation, use the reverse order of operations. Add or subtract first, and then multiply or divide.

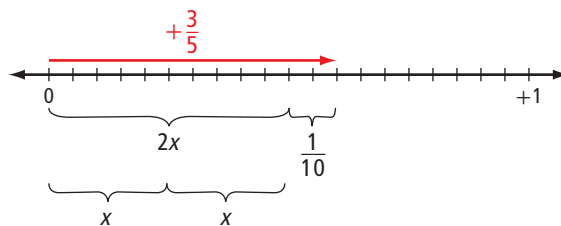
$$\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2}$$

Check by modelling the equation $2x + \frac{1}{10} = \frac{3}{5}$ on a number line.



This diagram models the original equation $2x + \frac{1}{10} = \frac{3}{5}$. How does it show that $2x = \frac{5}{10}$?

Now show the value of x .



The second diagram shows that $x = \frac{5}{20}$ or $\frac{1}{4}$.

The solution, $x = \frac{1}{4}$, is correct.

$$\text{b) } \frac{k}{3} - \frac{1}{2} = -1\frac{3}{4}$$

$$\frac{k}{3} - \frac{1}{2} = -\frac{7}{4}$$

You may prefer to work with integers than to perform fraction operations. Change from fractions to integers by multiplying by a common multiple of the denominators.

$$12 \times \frac{k}{3} - 12 \times \frac{1}{2} = 12 \times \left(-\frac{7}{4}\right)$$

$$4k - 6 = -21$$

$$4k - 6 + 6 = -21 + 6$$

$$4k = -15$$

$$\frac{4k}{4} = \frac{-15}{4}$$

$$k = -\frac{15}{4}$$

A common multiple of the denominators 3, 2, and 4 is 12.

Check:

$$\text{Left Side} = \frac{k}{3} - \frac{1}{2}$$

$$\text{Right Side} = -1\frac{3}{4}$$

$$= -\frac{15}{4} \div 3 - \frac{1}{2}$$

$$= \frac{-15}{4} \times \frac{1}{3} - \frac{1}{2}$$

$$= \frac{-5}{4} - \frac{1}{2}$$

$$= \frac{-5}{4} - \frac{2}{4}$$

$$= \frac{-7}{4} \text{ or } -1\frac{3}{4}$$

Left Side = Right Side

The solution, $k = -\frac{15}{4}$, is correct.

Show You Know

Solve and check.

$$\text{a) } 2y + \frac{1}{2} = \frac{3}{4}$$


$$\text{b) } \frac{n}{2} - \frac{3}{4} = 2\frac{3}{8}$$

Example 2: Solve Two-Step Equations With Decimals

Solve $\frac{a}{2.8} - 2.5 = -3.7$ and check the solution.

Solution

$$\begin{aligned}\frac{a}{2.8} - 2.5 &= -3.7 \\ \frac{a}{2.8} - 2.5 + 2.5 &= -3.7 + 2.5 \\ \frac{a}{2.8} &= -1.2 \\ 2.8 \times \frac{a}{2.8} &= 2.8 \times (-1.2) \\ a &= -3.36\end{aligned}$$

$$2.8 \times (-1.2) \approx 3 \times (-1) \approx -3$$


Check:

$$\begin{aligned}\text{Left Side} &= \frac{a}{2.8} - 2.5 & \text{Right Side} &= -3.7 \\ &= \frac{-3.36}{2.8} - 2.5 \\ &= -1.2 - 2.5 \\ &= -3.7\end{aligned}$$

Left Side = Right Side

The solution, $a = -3.36$, is correct.

Show You Know

Solve $\frac{h}{1.6} + 3.3 = 1.8$ and check the solution.

Example 3: Apply Two-Step Equations With Decimals

Colin has a long-distance telephone plan that charges 5¢/min for long-distance calls within Canada. There is also a monthly fee of \$4.95. One month, Colin's total long-distance charges were \$18.75. How many minutes of long-distance calls did Colin make that month?

Solution

Let m represent the unknown number of minutes.

The cost per minute is 5¢ or \$0.05.

The cost of the phone calls, in dollars, is $0.05m$.

The total cost for the month is the cost of the calls plus the monthly fee, or $0.05m + 4.95$.

The total cost for the month is \$18.75.



An equation that represents the situation is $0.05m + 4.95 = 18.75$.

$$0.05m + 4.95 - 4.95 = 18.75 - 4.95$$


$$0.05m = 13.80$$

$$\frac{0.05m}{0.05} = \frac{13.80}{0.05}$$

$$m = 276$$

$$\frac{13.80}{0.05} \approx \frac{15}{0.05}$$

$$\approx 300$$



Colin made 276 min of long-distance calls that month.

Check:

The cost for 276 min at 5¢/min is $\$0.05 \times 276$.

$$0.05 \times 276 = 13.80$$

The total cost for the month is $\$13.80 + \4.95 , which equals $\$18.75$.

This total cost agrees with the value stated in the problem.

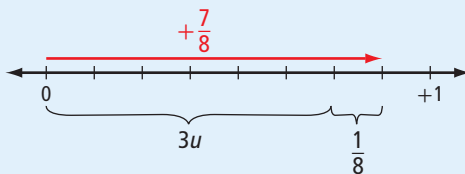
Show You Know

Suppose that Colin changes to a cheaper long-distance plan. This plan charges 4¢/min for long-distance calls within Canada, plus a monthly fee of \$3.95. For how many minutes could he call long distance in a month for the same total long-distance charge of \$18.75?

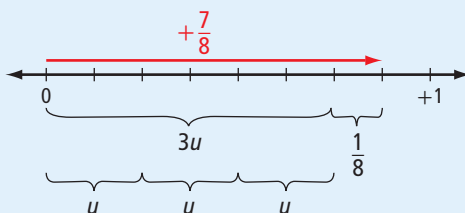
Key Ideas

- You can determine or check some solutions by using a model.

$$3u + \frac{1}{8} = \frac{7}{8}$$



$$3u = \frac{6}{8}$$



$$u = \frac{2}{8} \text{ or } \frac{1}{4}$$

- To isolate the variable in a two-step equation, use the reverse order of operations. Add or subtract first, and then multiply or divide.

$$0.4w - 1.5 = 0.3$$

$$0.4w - 1.5 + 1.5 = 0.3 + 1.5$$

$$0.4w = 1.8$$

$$\frac{0.4w}{0.4} = \frac{1.8}{0.4}$$

$$w = 4.5$$

- To solve two-step equations involving fractions, you may prefer to rewrite the equation and work with integers than to perform fraction operations.

$$\frac{w}{5} - \frac{3}{2} = \frac{1}{10}$$

To work with integers, multiply all terms by a common multiple of the denominators. For the denominators 5, 2, and 10, a common multiple is 10.

$$\begin{aligned} 10 \times \frac{w}{5} - 10 \times \frac{3}{2} &= 10 \times \frac{1}{10} \\ 2w - 15 &= 1 \\ 2w - 15 + 15 &= 1 + 15 \\ 2w &= 16 \\ \frac{2w}{2} &= \frac{16}{2} \\ w &= 8 \end{aligned}$$

- You can check solutions by using substitution.

$$\begin{aligned} \text{Left Side} &= \frac{w}{5} - \frac{3}{2} & \text{Right Side} &= \frac{1}{10} \\ &= \frac{8}{5} - \frac{3}{2} \\ &= \frac{16}{10} - \frac{15}{10} \\ &= \frac{1}{10} \end{aligned}$$

Left Side = Right Side

The solution, $w = 8$, is correct.

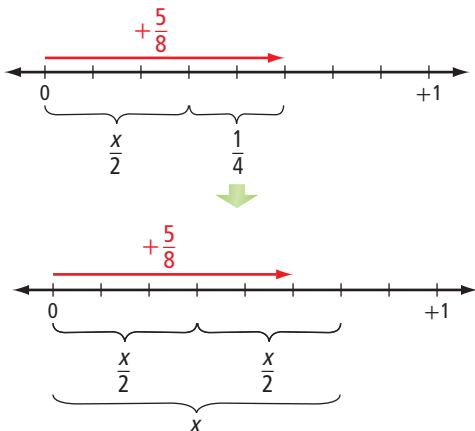
- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.

Check Your Understanding

Communicate the Ideas

- Explain how the diagrams model the equation

$$\frac{x}{2} + \frac{1}{4} = \frac{5}{8} \text{ and its solution. What is the solution?}$$



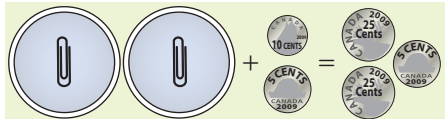
2. Ryan solved $2r + 0.3 = 0.7$ as follows. Do you agree with his solution? Explain.

$$\begin{aligned}\frac{2r}{2} + 0.3 &= \frac{0.7}{2} \\ r + 0.3 &= 0.35 \\ r + 0.3 - 0.3 &= 0.35 - 0.3 \\ r &= 0.05\end{aligned}$$

3. Jenna did not want to perform fraction operations to solve the equation $\frac{x}{2} - \frac{1}{9} = \frac{5}{6}$, so she first multiplied both sides by 54. Is this the common multiple you would have chosen? Explain.
4. a) Milos solved $0.05x - 0.12 = 0.08$ by multiplying both sides by 100 and then solving $5x - 12 = 8$. Show how he used this method to determine the correct solution.
- b) When Milos was asked to solve $\frac{x}{0.05} - 0.12 = 0.08$, he reasoned that he could determine the correct solution by solving $\frac{x}{5} - 12 = 8$. Do you agree with his reasoning? Explain.

Practise

5. Write an equation that is modelled by the following. Then, solve it.



6. Model the equation $3x + 0.14 = 0.50$ using concrete materials. Solve using your model.

For help with #7 and #8, refer to Example 1 on pages 306–307.

7. Solve.

$$\begin{array}{ll}\text{a) } 4y - \frac{2}{5} = \frac{3}{5} & \text{b) } 2d - \frac{1}{2} = \frac{5}{4} \\ \text{c) } \frac{n}{2} + 1\frac{2}{3} = \frac{1}{6} & \text{d) } \frac{4}{5} - 2\frac{1}{2}r = \frac{3}{10}\end{array}$$

8. Solve.

$$\begin{array}{ll}\text{a) } 1\frac{1}{2} = 4h + \frac{2}{3} & \text{b) } \frac{4}{3}x + \frac{3}{4} = \frac{1}{2} \\ \text{c) } \frac{3}{4} - \frac{d}{3} = \frac{3}{8} & \text{d) } -4\frac{2}{5} = -3\frac{1}{5} + \frac{7}{10}g\end{array}$$

For help with #9 and #10, refer to Example 2 on page 308.

9. Solve and check.

$$\begin{array}{l}\text{a) } \frac{x}{0.6} + 2.5 = -1 \\ \text{b) } 0.38 = 6.2 - \frac{r}{1.2}\end{array}$$

10. Solve.

$$\begin{array}{l}\text{a) } -0.02 - \frac{n}{3.7} = -0.01 \\ \text{b) } \frac{k}{-0.54} + 0.67 = 3.47\end{array}$$

For help with #11 and #12, refer to Example 3 on pages 308–309.

11. Solve and check.

$$\begin{array}{l}\text{a) } 2 + 12.5v = 0.55 \\ \text{b) } -0.77 = -0.1x - 0.45\end{array}$$

12. Solve.

$$\begin{array}{l}\text{a) } 0.074d - 3.4 = 0.707 \\ \text{b) } 67 = 5.51 + 4.3a\end{array}$$

Apply

- 13.** The cost of a pizza is \$8.50 plus \$1.35 per topping. How many toppings are on a pizza that costs \$13.90?
- 14.** Hiroshi paid \$34.95 to rent a car for a day, plus 12¢ for each kilometre he drove. The total rental cost, before taxes, was \$55.11. How far did Hiroshi drive that day?
- 15.** On Saturday morning, Marc had a quarter of his weekly allowance left. He spent a total of \$6.50 on bus fares and a freshly squeezed orange juice on Saturday afternoon. He then had \$2.25 left. How much is his weekly allowance?
- 16.** Nadia has a summer job in an electronics store. She is paid \$400 per week, plus 5% commission on the total value of her sales.
- One week, when the store was not busy, Nadia earned only \$510.30. What was the total value of her sales that week?
 - Nadia's average earnings are \$780 per week. What is the average value of her weekly sales?



- 17.** Benoit was helping his family build a new fence along one side of their yard. The total length of the fence is 28 m. They worked for two days and completed an equal length of fence on each day. On the third day, they completed the remaining 4.8 m of fence. What length of fence did they build on each of the first two days?
- 18.** The perimeter of a regular hexagon is 3.04 cm less than the perimeter of a regular pentagon. The perimeter of the regular hexagon is 21.06 cm. What is the side length of the regular pentagon?

- 19.** The greatest average annual snowfall in Alberta is on the Columbia Icefield. The greatest average annual snowfall in Manitoba is at Island Lake. An average of 642.9 mm of snow falls on the Columbia Icefield in a year. This amount of snow is 22.5 mm less than twice the annual average at Island Lake. What is the average annual snowfall at Island Lake?



Columbia Icefield

- 20.** During a camping trip, Nina was making a lean-to for sleeping. She cut a 2.5-m long post into two pieces, so that one piece was 26 cm longer than the other. What was the length of each piece?
- 21.** The average monthly rainfall in Victoria in July is 2.6 mm less than one fifth of the amount of rain that falls in Edmonton in the same period. Victoria averages 17.6 mm of rainfall in July. What is the average monthly rainfall in Edmonton in July?
- 22.** The temperature in Winnipeg was 7 °C and was falling by 2.5 °C/h. How many hours did it take for the temperature to reach -5.5 °C?
- 23.** Max and Sharifa are both saving to buy the same model of DVD player, which costs \$99, including tax. Max already has \$31.00 and decides to save \$8.50 per week from now on. Sharifa already has \$25.50 and decides to save \$10.50 per week from now on. Who can pay for the DVD player first? Explain.

24. A cylindrical storage tank that holds 375 L of water is completely full. A pump removes water at a rate of 0.6 L/s. For how many minutes must the pump work until 240 L of water remain in the tank?

25. The average distance of Mercury from the sun is 57.9 million kilometres. This distance is 3.8 million kilometres more than half the average distance of Venus from the sun. What is the average distance of Venus from the sun?

26. Create an equation of the form $\frac{x}{a} + b = c$ with each given solution. Compare your equations with your classmates' equations.

a) $\frac{2}{3}$

b) -0.8

27. Write a word problem that can be solved using an equation of the form $ax + b = c$. Include at least one decimal or fraction. Have a classmate solve your problem.

Extend

28. Solve.

a) $\frac{3}{2} + \frac{w}{4} = \frac{5}{6} - \frac{1}{2}$

b) $\frac{3}{4} \left(-\frac{2}{9} \right) = 4\frac{1}{2}x + \frac{1}{3}$

29. Solve. Express each solution to the nearest hundredth.

a) $0.75 + 0.16y + 0.2y = 0.34$

b) $\frac{-1.85}{0.74} = 2.22 - 0.57s$

30. Solve.

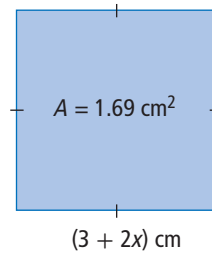
a) $\frac{0.2}{x} + 0.8 = 1.2$

b) $\frac{1}{2} - \frac{4}{n} = -\frac{1}{4}$

c) $\frac{-3.52}{h} - 1.31 = 1.19$

d) $4\frac{5}{6} = 3\frac{1}{3} - \frac{3}{y}$

31. Determine the value of x .



32. A freight train passes through a 750-m long tunnel at 50 km/h. The back of the train exits the tunnel 1.5 min after the front of the train enters it. What is the length of the train, in metres?



Math Link

A slice of canned corned beef contains about 0.21 g of sodium. This much sodium is 0.01 g more than the mass of sodium in four slices of roast beef. What is the mass of sodium in a slice of roast beef?

- Write an equation that models the situation.
- Solve the equation in two different ways.
- Which of your solution methods do you prefer? Explain.