Solving Equations: $ax = b, \dfrac{x}{a} = b, \dfrac{a}{x} = b$

Steve Nash is arguably the most successful basketball player that Canada has produced. He grew up in Victoria, British Columbia. In high school, he led his team to the provincial championship and was the province's player of the year. He has since become one of the top players in the National Basketball Association (NBA).

One year, Steve scored 407 points for the Phoenix Suns in 20 playoff games. If the equation $20x = 407$ represents this situation, what does x represent? What operation could you use to determine the value of x?

Did You Know?

A Canadian, Dr. James Naismith, invented the game of basketball in 1891. At the time, he was teaching in Springfield, Massachusetts.
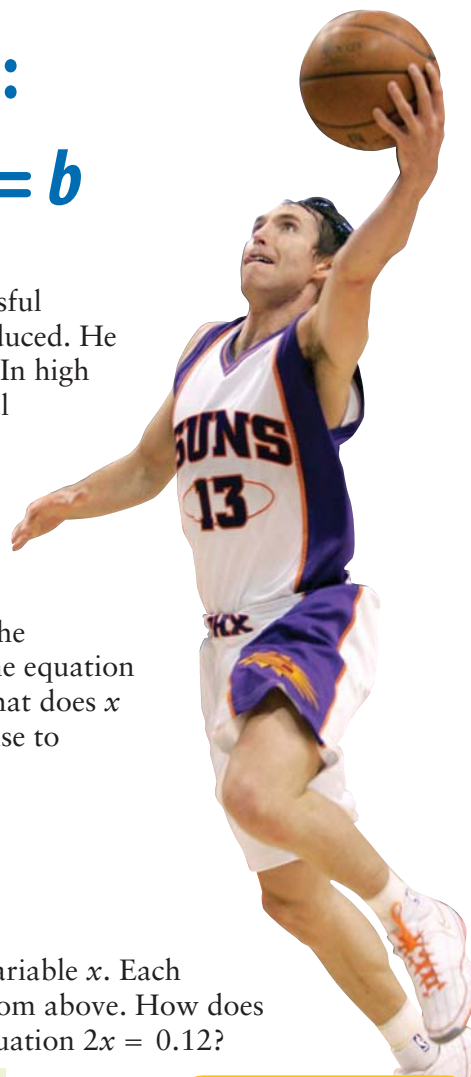
WWW Web Link

To learn more about Steve Nash's life, his career, and his work in communities, go to www.mathlinks9.ca and follow the links.

Materials

- coins
- paper cups or small containers
- paper clips

Explore Equations Involving Multiplication and Division

1. a) Each paper clip represents the variable x. Each circle represents a cup viewed from above. How does the diagram model the linear equation $2x = 0.12$?
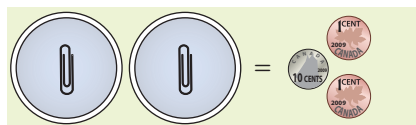
What other combination of coins could you use to represent 0.12?

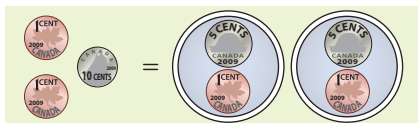**b)** How does the diagram model the solution to the equation in part a)? What is the solution?
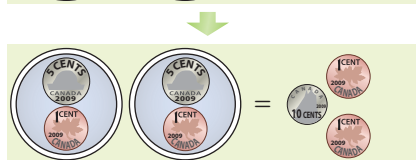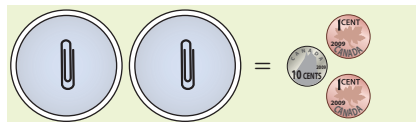
c) Explain how the second part of the diagram in part b) can also model the equation $0.06y = 0.12$. What is the solution? Explain.

2. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $3x = 0.6$

b) $0.05y = 0.25$

3. a) How does reversing the second part of the diagram from #1b) model the solution to the equation $\frac{n}{2} = 0.06$?

b) What is the solution? Explain.

c) Describe how the diagram in part a) can also model the solution to the equation $\frac{0.12}{k} = 0.06$. What is the solution? Explain.

4. Work with a partner to explore how to model the solutions to the following equations using manipulatives or diagrams. Share your models with other classmates.

a) $\frac{x}{3} = 0.05$

b) $\frac{0.33}{c} = 0.11$

Reflect and Check

5. a) How can you model solutions to equations of the form $ax = b$, $\frac{x}{a} = b$, and $\frac{a}{x} = b$ using manipulatives or diagrams?

b) Think of other ways to model the solutions. Explain how you would use them.

6. a) When a basketball player takes the ball away from an opposing player, it is called a *steal*. In his first nine seasons in the NBA, Steve Nash averaged 0.8 steals per game. Write and solve an equation that can be used to determine how many games it took, on average, for Steve to achieve four steals.

b) Use at least one other method to solve the problem. Share your solutions with your classmates.

Link the Ideas

Example 1: Solve One-Step Equations With Fractions

Solve each equation.

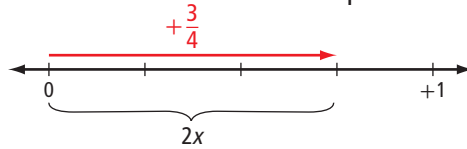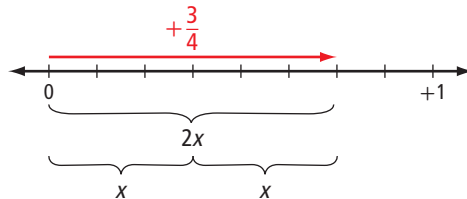**a)** $2x = \dfrac{3}{4}$ **b)** $\dfrac{m}{3} = -\dfrac{2}{5}$ **c)** $-2\dfrac{1}{2}k = -3\dfrac{1}{2}$

Solution

a) You can solve the equation $2x = \dfrac{3}{4}$ using a diagram or algebraically.

Strategies

Draw a Diagram

Method 1: Use a Diagram

Model the equation $2x = \dfrac{3}{4}$ on a number line.

The length of the curly bracket represents $2x$, so half of this length represents x.

> Why were the divisions on the number line changed from quarters to eighths?

The diagram shows that $x = \dfrac{3}{8}$.

Method 2: Solve Algebraically

Solve by applying the opposite operation.

$$2x = \dfrac{3}{4}$$
$$2x \div 2 = \dfrac{3}{4} \div 2$$
$$x = \dfrac{3}{4} \times \dfrac{1}{2}$$
$$= \dfrac{3}{8}$$

> Why do you divide both sides by 2?

> To divide $\dfrac{3}{4}$ by 2, why do you multiply by $\dfrac{1}{2}$?

Literacy Link

An *opposite operation* "undoes" another operation. Examples of opposite operations are:

• addition and subtraction
• multiplication and division

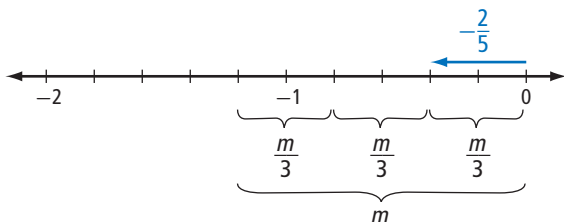Opposite operations are also called *inverse operations*.

Check:

Left Side $= 2x$ Right Side $= \dfrac{3}{4}$

$$= 2\left(\dfrac{3}{8}\right)$$
$$= \dfrac{6}{8}$$
$$= \dfrac{3}{4}$$

Left Side $=$ Right Side

The solution, $x = \dfrac{3}{8}$, is correct.

b) You can solve the equation $\frac{m}{3} = -\frac{2}{5}$ using a diagram or algebraically.

Method 1: Use a Diagram

Model the equation $\frac{m}{3} = -\frac{2}{5}$ on a number line.

$$-\frac{2}{5}$$
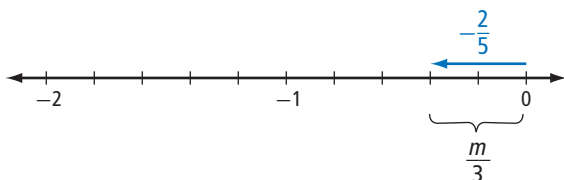
The length of the curly bracket represents $\frac{m}{3}$, so use three of these to represent m.

$$-\frac{2}{5}$$

The diagram shows that $m = -\frac{6}{5}$, or $-1\frac{1}{5}$.

Method 2: Solve Algebraically

Solve by applying the opposite operation.

$$\frac{m}{3} = -\frac{2}{5}$$

> What is the opposite operation of dividing by 3?

$$3 \times \frac{m}{3} = 3 \times \left(-\frac{2}{5}\right)$$

$$m = \frac{3}{1} \times \frac{-2}{5}$$

$$= \frac{-6}{5} \text{ or } -\frac{6}{5} \text{ or } -1\frac{1}{5}$$

> **M E**
> $3 \times \left(-\frac{2}{5}\right)$ $3 \times \left(-\frac{1}{2}\right)$
> $-1\frac{1}{2}$

Check:

Left Side $= \frac{m}{3}$ Right Side $= -\frac{2}{5}$

> $-1\frac{1}{5}$ is close to the estimate, so this answer is reasonable.

$$= \frac{-\frac{6}{5}}{3}$$

$$= -\frac{6}{5} \div 3$$

$$= \frac{-6}{5} \times \frac{1}{3}$$

$$= \frac{-6}{15}$$

$$= \frac{-2}{5} \text{ or } -\frac{2}{5}$$

Left Side $=$ Right Side

The solution, $m = -\frac{6}{5}$, is correct.

c) Isolate the variable by applying the opposite operation.

$$-2\frac{1}{2}k = -3\frac{1}{2}$$

$$-2\frac{1}{2}k \div \left(-2\frac{1}{2}\right) = -3\frac{1}{2} \div \left(-2\frac{1}{2}\right)$$

$$k = -\frac{7}{2} \div \left(-\frac{5}{2}\right)$$

$$= \frac{-7}{2} \div \frac{-5}{2}$$

$$= \frac{-7}{-5}$$

$$= \frac{7}{5} \text{ or } 1\frac{2}{5}$$

> What is the sign of the quotient when a negative is divided by a negative?

> $-4 \div (-3) = \frac{4}{3}$ **M** **E**

> One way to divide fractions with the same denominator is to simply divide the numerators. How else could you divide these fractions?

Check:

Left Side $= -2\frac{1}{2}k$ Right Side $= -3\frac{1}{2}$

$$= -2\frac{1}{2}\left(1\frac{2}{5}\right)$$

$$= -\frac{5}{2}\left(\frac{7}{5}\right)$$

$$= -\frac{35}{10}$$

$$= -\frac{7}{2} \text{ or } -3\frac{1}{2}$$

Left Side $=$ Right Side

The solution, $k = \frac{7}{5}$, is correct.

Show You Know

Solve.

a) $3x = -\frac{2}{3}$

b) $\frac{x}{2} = \frac{5}{6}$

c) $-1\frac{1}{4}y = 1\frac{3}{4}$

Example 2: Solve One-Step Equations With Decimals

Solve each equation. Check each solution.

a) $-1.2x = -3.96$ **b)** $\dfrac{r}{0.28} = -4.5$

Solution

a) Solve by applying the opposite operation.

$$-1.2x = -3.96$$

$$\frac{-1.2x}{-1.2} = \frac{-3.96}{-1.2}$$

$$x = 3.3$$

$-4 \div -1 = 4$ **M E**

C 3.96 $+\!\mathcal{C}\!-$ $\div$ 1.2 $+\!\mathcal{C}\!-$ **=** 3.3

Check:

Left Side $= -1.2x$ Right Side $= -3.96$
$= -1.2(3.3)$
$= -3.96$ **C** 1.2 $+\!\mathcal{C}\!-$ $\times$ 3.3 **=** -3.96
Left Side = Right Side

The solution, $x = 3.3$, is correct.

b) Isolate the variable by applying the opposite operation.

$$\frac{r}{0.28} = -4.5$$

$$0.28 \times \frac{r}{0.28} = 0.28 \times (-4.5)$$

$$r = -1.26$$

$0.28 \times 4.5 \quad 0.3 \times 4$ **M E**
1.2

C 0.28 $\times$ 4.5 $+\!\mathcal{C}\!-$ **=** -1.26

> What is the sign of the product when a positive is multiplied by a negative?

Check:

Left Side $= \dfrac{r}{0.28}$ Right Side $= -4.5$

$= \dfrac{-1.26}{0.28}$ **C** 1.26 $+\!\mathcal{C}\!-$ $\div$ 0.28 **=** -4.5

$= -4.5$
Left Side = Right Side

The solution, $r = -1.26$, is correct.

Show You Know

Solve and check.

a) $\dfrac{u}{1.3} = 0.8$

b) $5.5k = -3.41$

Example 3: Apply Equations of the Form $\frac{a}{x} = b$

The formula for speed is $s = \frac{d}{t}$, where s is speed, d is distance, and t is time. The length of a Canadian football field, including the end zones, is 137.2 m. If a horse gallops at 13.4 m/s, how much time would it take the horse to gallop the length of the field? Express your answer to the nearest tenth of a second.

Solution

Substitute the known values into the formula.

$$s = \frac{d}{t}$$

$$13.4 = \frac{137.2}{t}$$

> Why do you multiply by t?

$$t \times 13.4 = t \times \frac{137.2}{t}$$

$$t \times 13.4 = 137.2$$

$$\frac{t \times 13.4}{13.4} = \frac{137.2}{13.4}$$

$$t \qquad 10.2$$

> $130 \div 13 = 10$ M E

C 137.2 ÷ 13.4 = 10.23880597

The horse would take approximately 10.2 s to gallop the length of the field.

Check:
For a word problem, check your answer by verifying that the solution is consistent with the information given in the problem.

Calculate the speed by dividing the distance, 137.2 m, by the time, 10.2 s.

C 137.2 ÷ 10.2 = 13.45098039

Because the time was not exactly 10.2 s, this calculated speed of about 13.45 m/s is not exactly the same as the speed of 13.4 m/s given in the problem. But since these speed values are close, the answer is reasonable.

Show You Know

If a musher and her dog-team average 23.5 km/h during a dogsled race, how long will it take to sled 50 km? Express your answer to the nearest tenth of an hour.

Example 4: Write and Solve Equations

Winter Warehouse has winter jackets on sale at 25% off the regular price. If a jacket is on sale for $176.25, what is the regular price of the jacket?

Solution

Let p represent the regular price of the jacket.

The sale price is 75% of the regular price. So, the sale price is $0.75p$.

> How do you know that the sale price is 75% of the regular price?

Since the sale price is $176.25, an equation that represents the situation is
$$0.75p = 176.25$$
$$\frac{0.75p}{0.75} = \frac{176.25}{0.75}$$
$$p = 235$$

$200 \div 1 = 200$ M E

C 176.25 ÷ 0.75 = 235.

The regular price of the jacket is $235.

Check:
The price reduction is 25% of $235.
$0.25 \times \$235 = \$58.75$

The sale price is $235 - $58.75.
$\$235 - \$58.75 = \$176.25$

The calculated sale price agrees with the value given in the problem, so the answer, $235, is correct.
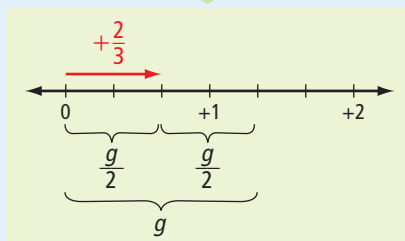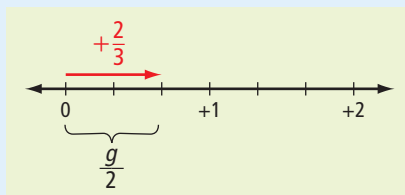
Show You Know

Winter Warehouse is selling mitts at 30% off the regular price. If the sale price is $34.99, what is the regular price of the mitts?

Key Ideas

- You can solve equations in various ways, including

using diagrams

$$\frac{g}{2} = \frac{2}{3}$$

using concrete materials

$$2x = 0.10$$

$$x = 0.05$$

$$g = \frac{4}{3} \text{ or } 1\frac{1}{3}$$

using an algebraic method

$$\frac{-1.4}{p} = -0.8$$

$$p \times \left(\frac{-1.4}{p}\right) = p \times (-0.8)$$

$$-1.4 = p \times (-0.8)$$

$$\frac{-1.4}{-0.8} = \frac{p \times (-0.8)}{-0.8}$$

$$1.75 = p$$

- You can check solutions by using substitution.

Left Side $= \dfrac{-1.4}{p}$ Right Side $= -0.8$

$$= \frac{-1.4}{1.75}$$

$$= -0.8$$

C 1.4 +⟳− ÷ 1.75 = -0.8

Left Side $=$ Right Side

The solution, $p = 1.75$, is correct.

- To check the solution to a word problem, verify that the solution is consistent with the facts given in the problem.
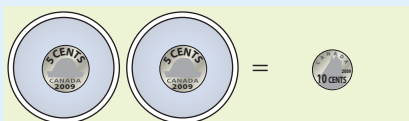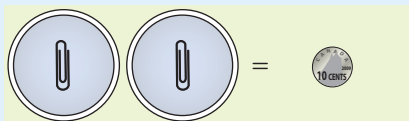
Check Your Understanding

Communicate the Ideas

1. To solve the equation $\frac{y}{2} = \frac{5}{3}$, John first multiplied both sides by 3.

   a) Do you think that John's first step is the best way to isolate the variable y? Explain.

   b) How would you solve the equation?

2. When Ming solved $0.3g = 0.8$, her value for g was $2.666\ldots$. She expressed this to the nearest tenth, or 2.7. When Ming checked by substitution, she found that the left side and the right side did not exactly agree.

   Left Side $= 0.3g$ Right Side $= 0.8$
             $= 0.3(2.7)$
             $= 0.81$

   a) How could Ming make the left side and right side agree more closely?

   b) Did Ming's check show that the solution was correct? Explain.

3. The length of Shamika's stride is 0.75 m. Both Amalia and Gustav were trying to calculate how many strides it would take Shamika to walk 30 m from her home to the bus stop.

   a) Amalia represented the situation with the equation $0.75p = 30$. Explain her thinking.

   b) Gustav represented the situation with the equation $\frac{30}{p} = 0.75$. Explain his thinking.

   c) Whose equation would you prefer to use? Explain.

Practise

4. Write an equation that is represented by the model shown. Then, solve it.
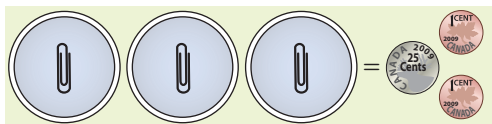
For help with #5 to #7, refer to Example 1 on pages 294–296.

5. Model the solution to the equation $4x = \frac{3}{4}$ using a number line.

6. Solve.

   a) $2v = \frac{-5}{6}$ **b)** $\frac{x}{2} = \frac{2}{5}$

   c) $\frac{4}{3} = -1\frac{1}{4}a$ **d)** $-1\frac{1}{2}x = -2\frac{1}{4}$

7. Solve.

   a) $\frac{3}{5} = \frac{x}{4}$ **b)** $2y = -\frac{6}{5}$

   c) $-\frac{7}{6} = -\frac{4}{3}n$ **d)** $2\frac{2}{3}w = 1\frac{1}{6}$

For help with #8 and #9, refer to Example 2 on page 297.

8. Solve and check.

   a) $-5.6x = 3.5$ **b)** $\frac{e}{-2.2} = -0.75$

9. Solve.

   a) $\frac{h}{4.1} = 3.6$ **b)** $1.472 = 0.46c$

10. Solve and check.

a) $-5.5 = \dfrac{1.1}{a}$ **b)** $\dfrac{4.8}{m} = 6.4$

11. Solve. Express each solution to the nearest hundredth.

a) $\dfrac{2.02}{n} = 0.71$ **b)** $-7.8 = \dfrac{-4.3}{x}$

Apply

12. The average speed of a vehicle, s, is represented by the formula $s = \dfrac{d}{t}$ where d is the distance driven and t is the time.

a) If Pablo drove at an average speed of 85 km/h for 3.75 h, what distance did he drive?

b) If Sheila drove 152 km at an average speed of 95 km/h, how much time did her trip take?

13. A roll of nickels is worth $2.00. Write and solve an equation to determine the number of nickels in a roll.

14. Write and solve an equation to determine the side length, s, of a square with a perimeter of 25.8 cm.

15. Without solving the equation $-\dfrac{5}{d} = -1.3$, predict whether d is greater than or less than 0. Explain your reasoning.

16. The diameter, d, of a circle is related to the circumference, C, by the formula $\dfrac{C}{d} = $. Calculate the diameter of a circle with a circumference of 54.5 cm. Express your answer to the nearest tenth of a centimetre.

17. A regular polygon has a perimeter of 34.08 cm and a side length of 5.68 cm. Identify the polygon.

> **⬤⬤ Literacy Link**
>
> A *regular polygon* has equal sides and equal angles. For example, a regular pentagon has five equal sides. Each angle measures 108°.
>
>
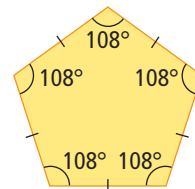
18. One year, a student council sold 856 copies of a school yearbook. Four fifths of the students at the school bought a copy. How many students did not buy a yearbook?

19. A score of 17 on a math test results in a mark of 68%. What score would give you a mark of 100%?

20. The area of Nunavut is about $4\dfrac{3}{8}$ times the area of the Yukon Territory. Nunavut covers 21% of Canada's area. What percent of Canada's area does the Yukon Territory cover?

Lewis Lake, Yukon

21. Dianne spends 40% of her net income on rent and 15% of her net income on food. If she spends a total of $1375 per month on rent and food, what is her net monthly income?

22. Ellen and Li play on the same basketball team. In one game, Ellen scored one tenth of the team's points and Li scored one fifth of the team's points. Together, Ellen and Li scored 33 points. How many points did the team score altogether?

23. Sailaway Travel has a last-minute sale on a Caribbean cruise at 20% off. Their advertisement reads, "You save $249.99." What is the sale price of the cruise?

24. Organizers of the Canadian Francophone Games hope to attract 500 volunteers to help host the games. The organizers predict that there will be about three and half times as many experienced volunteers as first-time volunteers. About how many first-time volunteers do they expect to attract?

25. A square piece of paper is folded in half to make a rectangle. The perimeter of the rectangle is 24.9 cm. What is the side length of the square piece of paper?

Extend

26. Solve.

a) $\dfrac{1}{3} + \dfrac{1}{6} = \dfrac{5}{6}x$

b) $-\dfrac{0.45}{1.8} = -\dfrac{0.81}{z}$

c) $\dfrac{y}{4} - \dfrac{y}{3} = -\dfrac{1}{10}$

d) $\dfrac{f}{0.55} = 2.6 - 3.5$

27. Solve. Express each solution to the nearest hundredth.

a) $0.75 + 1.23 = -3.9t$

b) $\dfrac{6.3}{h} = 2(-4.05)$

28. Solve and check.

a) $x \div \dfrac{1}{2} = -\dfrac{3}{4}$

b) $t \div \left(-\dfrac{2}{3}\right) = -\dfrac{1}{2}$

c) $\dfrac{5}{6} \div y = \dfrac{2}{3}$

d) $-\dfrac{2}{5} \div g = \dfrac{3}{10}$

29. a) A jar contains equal numbers of nickels and dimes. The total value of the coins is $4.05. How many coins are in the jar?

b) A jar contains a mixture of nickels and dimes worth a total of $4.75. There are three times as many nickels as dimes. How many dimes are there?

30. A cyclist is travelling six times as fast as a pedestrian. The difference in their speeds is 17.5 km/h. What is the cyclist's speed?

Math Link

Solve parts a) and b) in at least two different ways. Write and solve an equation as one of the methods for each part. Share your solutions with your classmates.

Three dried figs contain about 1.2 mg of iron.

a) What is the mass of iron in one dried fig?

b) Teenagers need about 12 mg of iron per day. How many dried figs would you have to eat to get your recommended daily amount of iron?

c) Write a formula that relates the mass of iron to the number of figs. Use your equation to calculate the mass of iron in eight figs.

d) Use your formula in part c) to determine the number of figs that contain 1.8 mg of iron.