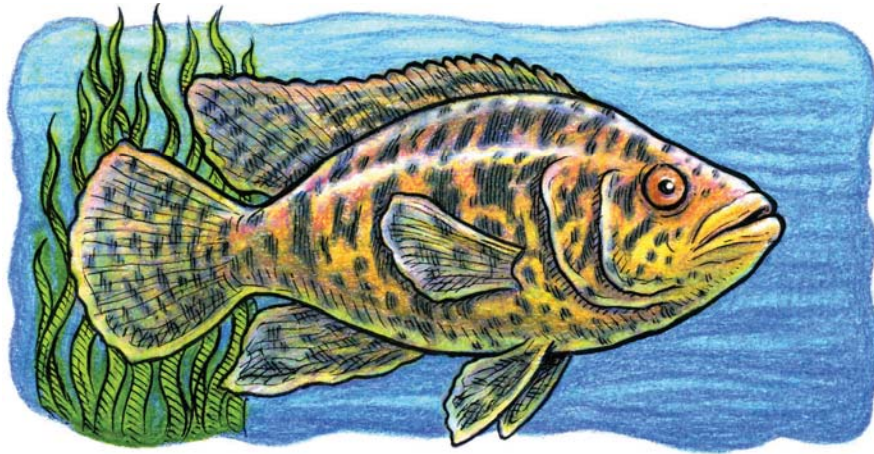# Dividing Polynomials by Monomials

## Focus on…

After this lesson, you will be able to…

• **divide a polynomial by a monomial**

When you are buying a fish tank, the size of the tank depends on the size and habits of the fish. A tank for a jaguar cichlid, or *Parachromis managuensis*, should have the minimum dimensions shown, in metres.

The volume of the rectangular tank can be represented by the polynomial expression $7.5w^2 - 3w$. How could you determine a polynomial expression that represents the length of the tank in terms of w?

0.6 m

w

Explore Dividing a Polynomial by a Monomial
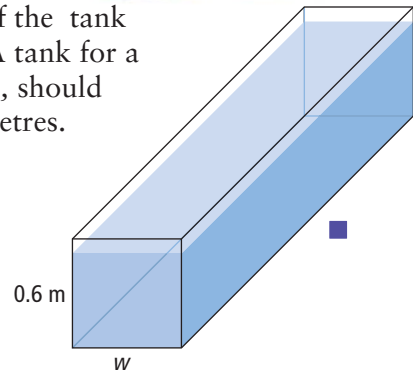
Materials

• algebra tiles

A rectangular solid has a width of $2x$, a height of 3, and an unknown length. The area of the base of the solid is represented by the polynomial $2x^2 + 4x$.

3

$2x$

1. Show that the volume of the solid shown can be represented by the polynomial $6x^2 + 12x$.

2. Use algebra tiles to represent the area of the rectangular base.

3. Count the number of x-tiles and 1-tiles required for the missing dimension of the rectangle. What expression represents the missing dimension?
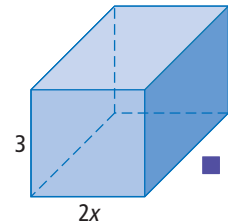
Reflect and Check

> What happens if you multiply your expression by the width of the rectangular solid?

4. Show that your expression for the missing dimension in #3 is correct.

5. Describe the steps you would take to find an expression for the ratio of the volume to the side measuring $2x$.

Link the Ideas

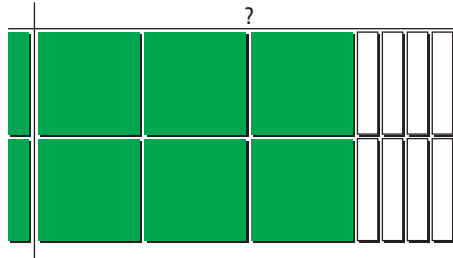## Example 1: Divide a Polynomial by a Monomial Using a Model

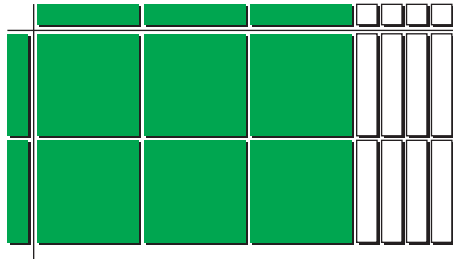Determine the quotient.

$$\frac{6x^2 - 8x}{2x}$$

Solution

You can use algebra tiles.
Use 6 positive x^2-tiles and
8 negative x-tiles to represent
the polynomial $6x^2 - 8x$.

The vertical side of the rectangle
represents the monomial divisor, $2x$.

Count the number of positive x-tiles
and negative 1-tiles required to
complete the horizontal side of the
rectangle.

There are 3 positive x-tiles and
4 negative 1-tiles, or $3x - 4$. This
expression represents the result of
dividing the polynomial, $6x^2 - 8x$,
by the monomial, $2x$.

$$\frac{6x^2 - 8x}{2x} = 3x - 4$$

Check:
Multiply the quotient, $3x - 4$, by the divisor, $2x$.

$(2x)(3x - 4)$
$= (2x)(3x) - (2x)(4)$
$= 6x^2 - 8x$

How do you know
that the answer is
correct?

You can also determine
the quotient
algebraically.

$$\frac{6x^2 - 8x}{2x}$$

$$= \frac{6x^2}{2x} - \frac{8x}{2x}$$

$$= \frac{\overset{3x}{\cancel{6x^2}}}{\underset{1}{\cancel{2x}}} - \frac{\overset{4}{\cancel{8x}}}{\underset{1}{\cancel{2x}}}$$

$$= 3x - 4$$

Show You Know

Determine each quotient.

a) $\dfrac{3x^2 + 6x}{3x}$

b) $\dfrac{8x^2 - 2x}{2x}$

Example 2: Dividing a Polynomial by a Monomial Algebraically

a) What is the ratio of the surface area to the radius of the cylinder? Write the ratio in simplified form.

b) If the height, h, of the cylinder is the same as the radius, r, what is the ratio of the surface area to the radius? Write the ratio in simplified form.
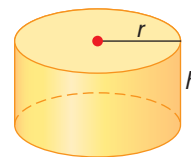
Solution

a) $\dfrac{\text{surface area}}{\text{radius}} = \dfrac{2\pi r^2 + 2\pi rh}{r}$

The expression can be broken down into two parts.

$$\dfrac{\text{surface area}}{\text{radius}}$$

$$= \dfrac{2\pi r^2}{r} + \dfrac{2\pi rh}{r}$$

$$= \dfrac{2\pi r\overset{r}{\cancel{r^2}}}{\underset{1}{\cancel{r}}} + \dfrac{2\pi \overset{1}{\cancel{r}}h}{\underset{1}{\cancel{r}}}$$

$$= 2\pi r + 2\pi h$$

b) Substitute $h = r$ into the ratio from part a).

$$\dfrac{\text{surface area}}{\text{radius}}$$

$$= 2\pi r + 2\pi h$$

$$= 2\pi r + 2\pi(r)$$

$$= 2\pi r + 2\pi r$$

$$= 4\pi r$$

Show You Know

Determine each quotient.

a) $\dfrac{15x^2 - 12x}{3x}$

b) $\dfrac{-2t^2 + 4t}{2t}$

Key Ideas

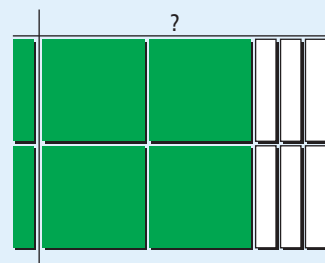- You can divide a polynomial by a monomial using a model.

  $\dfrac{4x^2 - 6x}{2x}$

  The unknown side length of the rectangle is made up of $2x - 3$ tiles.

  $\dfrac{4x^2 - 6x}{2x} = 2x - 3$

- When you divide a polynomial by a monomial algebraically, you can divide the numerical coefficients and apply the exponent laws to the variables.

  $\dfrac{4x^2 - 8x}{2x}$

  $= \dfrac{4x^2}{2x} - \dfrac{8x}{2x}$

  $= \dfrac{\overset{2x}{\cancel{4x^2}}}{\underset{1}{\cancel{2x}}} - \dfrac{\overset{4}{\cancel{8x}}}{\underset{1}{\cancel{2x}}}$

  $= 2x - 4$

Check Your Understanding

Communicate the Ideas

1. Explain how you would perform the following division: $\dfrac{3x^2 + 6x}{2x}$.

2. Anita used the following method to simplify an expression:

$$\frac{9k^2 - 3k}{3}$$
$$= \frac{9k^2}{3} - \frac{3k}{3}$$
$$= 3k - 1$$

    a) Show that Anita's solution is incorrect.

    b) How would you correct her solution?

3. Use a model to show a polynomial division statement with a quotient of $3x + 2$.

Practise

For help with #4 to #7, refer to Example 1 on page 273.

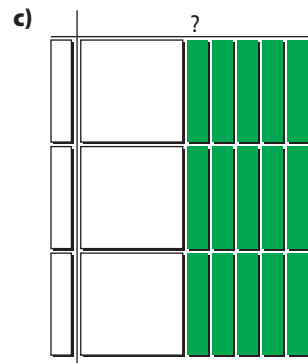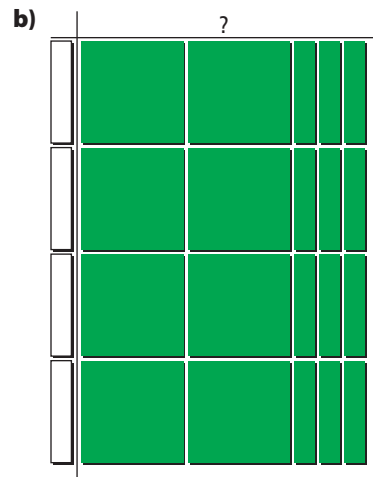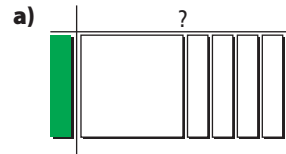**4.** What division statement is represented by the algebra tiles? Determine the quotient.

    a)

    b)

    c)
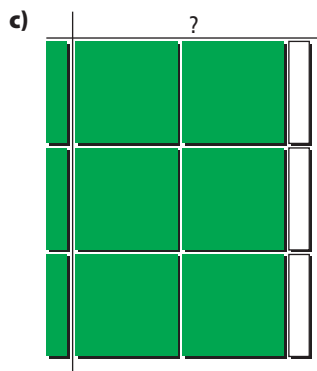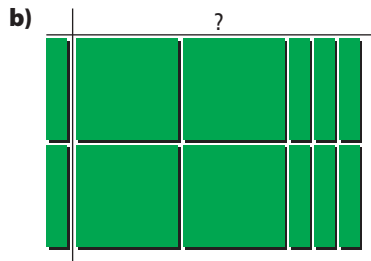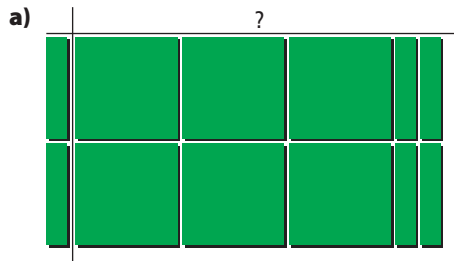
5. Determine the division statement represented by the algebra tiles and give the quotient.

    a)

    b)

    c)

6. Divide each expression, using a model.

a) $\dfrac{5x^2 - 10x}{5x}$

b) $\dfrac{4x^2 + 12x}{2x}$

7. Use a model to divide each expression.

a) $\dfrac{-8x^2 - 4x}{4x}$

b) $\dfrac{-3x^2 + 5x}{-x}$

For help with #8 and #9, refer to Example 2 on page 274.

8. Divide.

a) $\dfrac{2y^2 + 4.2y}{2y}$

b) $\dfrac{12m^2 - 6.2m + 24}{2}$

c) $\dfrac{-18y^2 - 6y}{-6y}$

d) $\dfrac{3cv - 2.7c}{3c}$

9. Determine each quotient.

a) $\dfrac{2.7c^2 + 3.6c}{3c}$
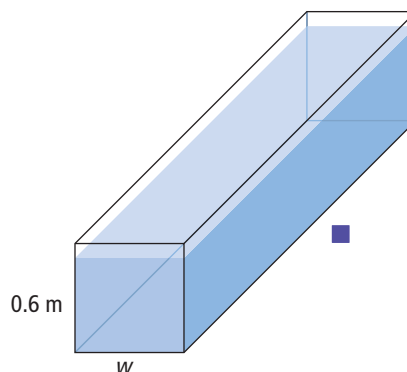
b) $\dfrac{2x^2 + 8xy}{x}$

c) $\dfrac{-s^2 - 1.5st}{5s}$

d) $\dfrac{-14w^2 - 7w + 0.5}{0.5}$

Apply

10. A dump truck holds 10 m³ of soil. You are filling a rectangular space in a yard with the dimensions of $(2x + 3)$ by $5x$ by 2, in metres. What polynomial expression represents the number of truck loads of soil you will need?

11. A rectangular fish tank has the dimensions shown, in metres. The volume of the tank can be represented by $7.5w^2 - 3w$.

a) What polynomial expression represents the area of the base of the tank?

b) What polynomial expression represents the length of the tank?

c) What is the length of the tank if the width is 0.6 m? What is the volume of the tank?

12. For their Valentine's Day dance, the grade 9 students want to decorate the end wall of the gym with red poster paper. The area of the wall is given by the polynomial $45x^2 + 20x$. One sheet of poster paper covers an area given by the monomial $5x$. What polynomial expression represents the number of sheets of paper the students will need to cover the wall?

13. A rectangle has an area of $9x^2 - 3x$ square units. The width of the rectangle is $3x$ units. What is the length?
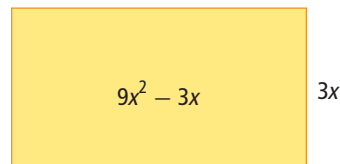
14. The formula used to predict the distance an object falls is $d = 4.9t^2 + vt$. In the formula, d is the distance, in metres, t is the time, in seconds, and v is the starting velocity of the object, in metres per second.

a) The average speed of a falling object is calculated as $s = \dfrac{d}{t}$, where s is the average speed, in metres per second. Use this information to develop a formula for the average speed of a falling object in terms of t and v.

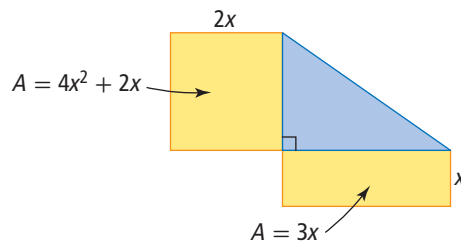**b)** What is the average speed of an object that falls for 5 s, if it starts from a resting position?

Extend

15. Divide.

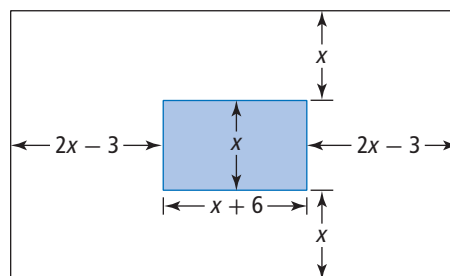**a)** $\dfrac{3.6gf + 0.93g}{0.3g}$

b) $\dfrac{\frac{2}{3}b^2 - \frac{1}{3}ab + \frac{1}{3}b}{\frac{1}{3}b}$

c) $\dfrac{-4.8x^2 + 3.6x - 0.4}{0.2}$

16. Two rectangles have common sides with a right triangle, as shown. The areas and widths of the rectangles are shown. What is a simplified expression for the area of the triangle?

17. What is the ratio of the area of the shaded rectangle to the area of the large rectangle?

18. If a rectangle has length $2xy$ and area $12x^2y + 6xy^2$, what is its perimeter?

Math Link

You are designing a park that includes a large parking lot that will be covered with gravel.

a) Design two different-shaped parking lots using any single shape or combination of regular shapes. Include the dimensions on a drawing of each parking lot design. Note that you will need to be able to calculate the area of your parking lots. Each area should be a different shape. Make them no less than 200 m² and no greater than 650 m².

b) A truck with dimensions similar to those shown in the picture will deliver the gravel. Write an expression for the approximate area that a single load of gravel will cover to a depth of 5 cm.

c) There are three sizes of trucks that can deliver the gravel. The widths are 1.5 m, 2 m, and 3 m. Approximately how many truckloads would it take for each truck size to deliver the required amount of gravel for each of your parking lots? You will cover each parking lot to a depth of 5 cm. Show your work.

d) Which truck size do you think would be the most efficient to use for each of your parking lots? Explain your reasoning.