7.2 Multiplying Polynomials by Monomials

Focus on...

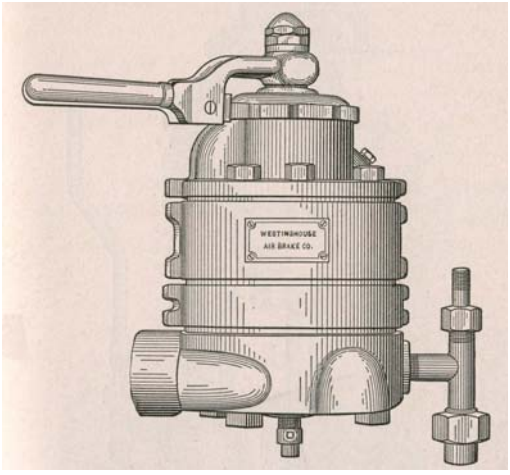
After this lesson, you will be able to...

- multiply a polynomial by a monomial

In the 1880s, braking a train was a tough job. The engineer "whistled down" for brakes and reversed his engine. Two people were responsible for the brakes. One rode in the cab on the engine and the other rode in the caboose. These people would run toward each other over the swaying, rocking car tops, tightening each car's brake control wheel as they went.

Physics Link

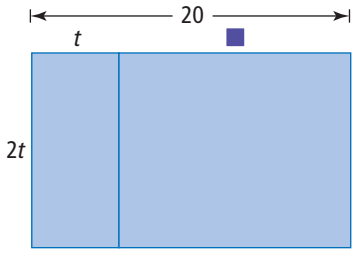
Air brakes on trains became commonplace in the 1900s, making the task of stopping a train much easier. Air brakes use compressed air in a piston to push the brake shoe onto the train wheel. The system works like a bicycle brake.



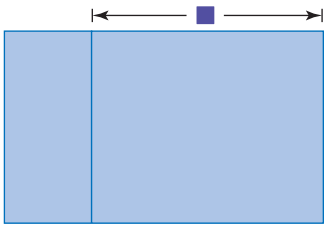
Explore Multiplying a Polynomial by a Monomial

When a train's brakes are applied, the train travels a distance before it stops. After t seconds, the distance, in metres, that the train travels is given by the polynomial $2t(20 - t)$.

1. What part of the diagram does $2t(20 - t)$ represent?



2. What polynomial represents the unknown length in the diagram? How did you determine this polynomial?



3. Find three rectangles in the diagram. What is an expression for the area of the largest rectangle? What is an expression for the area of the smallest rectangle?

4. What is the difference in area between the largest and smallest rectangles? Show two ways to find your answer.

5. Calculate the area of the medium-sized rectangle using the dimension you determined in #2.

Reflect and Check

6. Describe the steps you used in #5 to calculate the area of the medium-sized rectangle.

7. How is the area of the medium-sized rectangle related to the areas of the large rectangle and the small rectangle?

8. How far does the train travel in 10 s? Show how you arrived at your answer.

Literacy Link

A polynomial is made up of terms connected by addition or subtraction.

Examples:
$x + 5$
$2d - 2.4$
x
$3s^2 + 5s - 6$
$\frac{h^2}{2} - \frac{h}{4}$

Link the Ideas

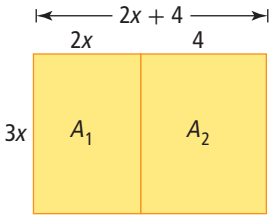
Example 1: Multiply a Polynomial by a Monomial Using an Area Model

Determine the product.
$(3x)(2x + 4)$

Strategies
Draw a Diagram

Solution

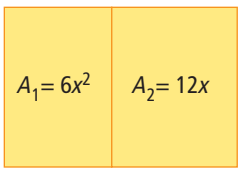
Draw a rectangle with side dimensions that represent $3x$ and $2x + 4$.



Calculate the area of each rectangle.
$A_1 = (3x)(2x)$
$A_1 = 6x^2$

$A_2 = (3x)(4)$
$A_2 = 12x$

The total area is $A_1 + A_2 = 6x^2 + 12x$.



Show You Know

Calculate each product.

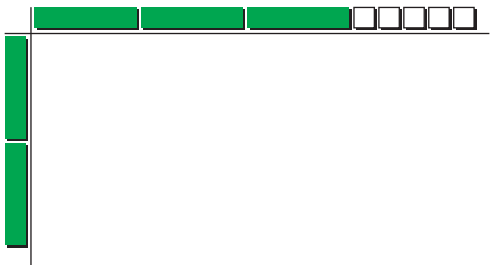
a) $(2x)(x + 3)$ **b)** $(2 + c)(c)$

Example 2: Multiply a Polynomial by a Monomial Using Algebra Tiles

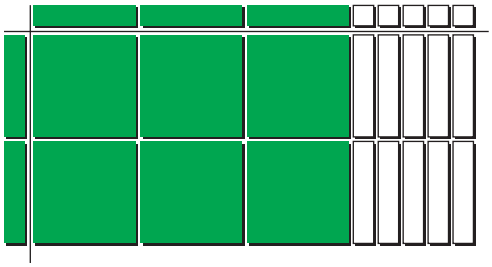
Find the product.
$(2x)(3x - 5)$

Solution

You can use x-tiles and negative 1-tiles to model $2x$ and $3x - 5$.



Use x^2-tiles and negative x-tiles to model $(2x)(3x - 5)$.



There are 6 positive x^2-tiles and 10 negative x-tiles.

So, $(2x)(3x - 5) = 6x^2 - 10x$.

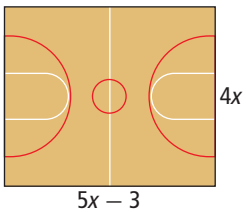
Show You Know

Find each product.

a) $(2 + 3x)(3x)$ **b)** $(4x)(2x - 1)$

Example 3: Multiply a Polynomial by a Monomial Algebraically

The dimensions of a rectangular gym floor are represented by the expressions $4x$ and $5x - 3$. What is a polynomial expression for the area of the gym floor? Write the expression in simplified form.



Solution

You can calculate the area of a rectangle, A, by multiplying the width by the length.

$A = (4x)(5x - 3)$

You can apply the distributive property. Multiply $4x$ by each term in the binomial $5x - 3$.

$A = (4x)(5x - 3)$
$A = (4x)(5x) - (4x)(3)$
$A = (4)(5)(x)(x) - (4)(3)(x)$
$A = 20x^2 - 12x$

A simplified expression for the area of the gym floor is $20x^2 - 12x$.

Literacy Link

The *distributive property* allows you to expand algebraic expressions. Multiply the monomial by each term in the polynomial.

$a(b + c) = ab + ac$

Literacy Link

A *binomial* is a polynomial with two terms, such as $6y^2 + 3$ and $2x - 5$.

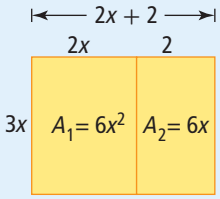
Show You Know

Calculate each product.

a) $(-3x)(2x + 5)$ **b)** $(5y)(11 - x)$

Key Ideas

- You can represent the multiplication of a polynomial by a monomial using models.
  - area model

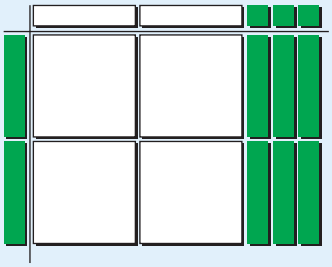


$(3x)(2x + 2)$

The product is represented by $A_1 + A_2$.

$(3x)(2x + 2) = 6x^2 + 6x$

  - algebra tiles



$(2x)(-2x + 3)$

There are 4 negative x^2-tiles and 6 positive x-tiles.

$(2x)(-2x + 3) = -4x^2 + 6x$

- To multiply a polynomial by a monomial algebraically, you can expand the expression using the distributive property. Multiply each term of the polynomial by the monomial.

$$(-1.2x)(3x - 7)$$
$$= (-1.2x)(3x) - (-1.2x)(7)$$
$$= -3.6x^2 + 8.4x$$

Check Your Understanding

Communicate the Ideas

1. Describe two methods you could use to multiply polynomials by monomials.

2. Sara is going to simplify the expression $(3x)(2x + 4)$. Can she add the terms in the brackets and then multiply? Explain.

3. Mahmoud used the following method to expand the expression $(5x)(2x + 1)$.

$(5x)(2x + 1) = 10x^2 + 1$

a) Show that Mahmoud's solution is incorrect.

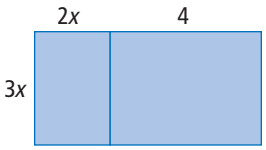
b) How would you correct his solution?

Practise

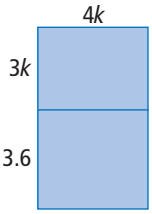
For help with #4 to #7, refer to Example 1 on page 266.

4. What multiplication statement is represented by each area model?

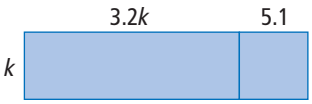
a)



b)



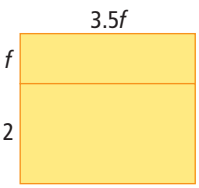
c)



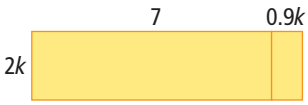
5. Determine the multiplication statement shown by each area model.

a)

b)



c)



6. Expand each expression using an area model.

a) $(3.2r + 1)(4r)$ **b)** $\left(\frac{1}{2}a\right)(3a + 6)$

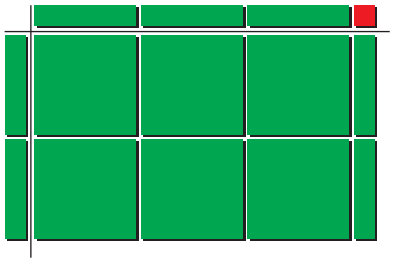
7. Use an area model to expand each expression.

a) $(2x)(4x + 2)$ **b)** $(6k + 2)(4.5k)$

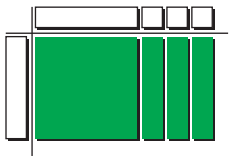
For help with #8 to #11, refer to Example 2 on pages 266-267.

8. What multiplication statement is represented by the algebra tiles?

a)



b)



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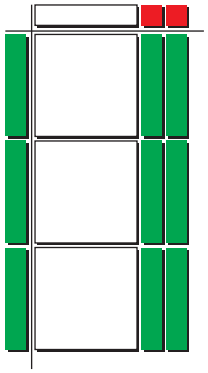
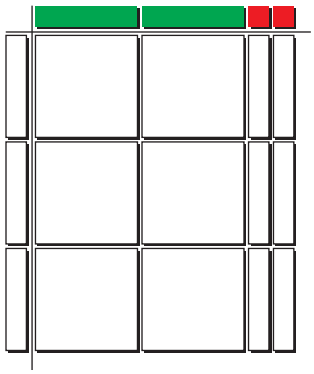
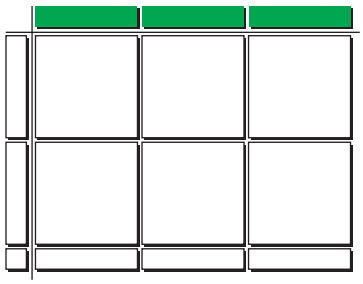


9. Determine the multiplication statement shown by the algebra tiles.

a)



b)



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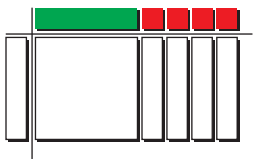


10. Expand each expression, using algebra tiles.

a) $(x - 5)(3x)$ **b)** $(2x)(-2x + 3)$

11. Use algebra tiles to expand each expression.

a) $(4x + 2)(-3x)$ **b)** $(-4x)(3x - 1)$

For help with #12 and #13, refer to Example 3 on page 267.

12. Expand using the distributive property.

a) $(2x)(3x - 1)$

b) $(3p)(2p - 0.8)$

c) $(0.5m)(7 - 12m)$

d) $\left(\frac{1}{2}r - 2\right)(-r)$

e) $(2n - 7)(8.2)$

f) $(3x)(x + 2y + 4)$

13. Multiply.

a) $(4j)(2j - 3)$

b) $(-1.2w)(3w - 7)$

c) $(6x)(4 - 2.4x)$

d) $\left(\frac{3}{7}v + 7\right)(-1)$

e) $(3 - 9y)(y)$

f) $(-8a - 7b - 2)(8a)$

Apply

14. A rectangular Kwakiutl button blanket has a width of $3x$ and a length of $4x - 3$.

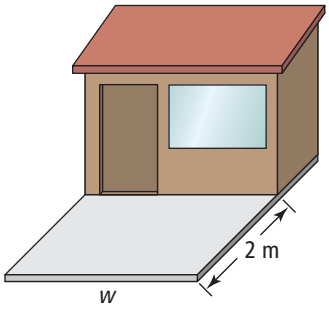
a) What is an expanded expression for the area of the blanket?

b) What is a simplified expression for the perimeter of the blanket?

15. Lee has decided to build a shed on a square concrete slab. The shed has the same width, w, as the slab. Its length is 2 m shorter than the width of the slab.

a) What is an expression for the area of the shed?

b) If the width, w, of the slab is 4 m, what is the area of the shed?



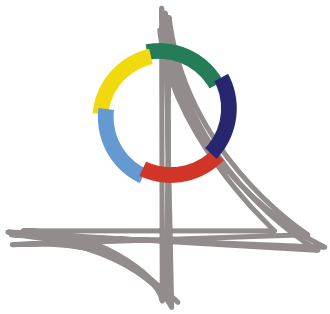
16. The basketball court for the Jeux de la Francophonie is 5.5 m longer than 1.5 times the width.

a) What is an expression for the area of the basketball court?

b) If the length is 28 m, what is the area of the basketball court?

Did You Know?

The Jeux de la Francophonie are games for French-speaking people. They are held every four years in different locations around the world. The games include sports and artistic events. Canada is represented by three teams: Québec, New Brunswick, and a third team representing the rest of Canada.



17. A rectangular field is $(4x + 2)$ m long. The width of the field is 2 m shorter than the length. What is an expression for the area of the field?

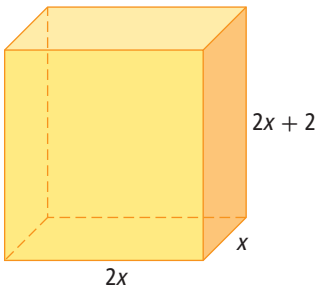
18. A rectangular skateboard park is $(3x)$ m long. Its width is 4 m less than the length.

a) What is an expression for the area of the park?

b) If $x = 15$, what is the area of the park?

Extend

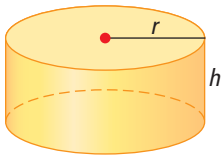
19. A rectangluar packing crate has the dimensions shown, in metres.



a) What is an expression for the total surface area of the crate?

b) What is an expression for the volume of the crate?

20. The surface area, SA, of a cylinder is $SA = 2\pi r^2 + 2\pi rh$, where r is the radius and h is the height. The formula for the volume, V, of a cylinder is $V = \pi r^2 h$. What is the surface area of a cylinder that has a height of 5 cm and a volume of 80π cm^3?



21. A rectangle measuring $(12n)$ m by 8 m has a square of side length 2 m cut out in the four corners. The cut-out shape forms an open box when the four corners are folded and taped.

a) Draw the box and label its dimensions.

b) What is the surface area of this open box?

c) What is the capacity of this box?

Math Link

You are drawing up plans for a landscape design. You are going to include one of the following design elements, which will be in the shape of a rectangle:

- swimming pool
- concrete patio
- hockey rink
- beach volleyball pit

The rectangular shape is 2 m longer than twice the width. You must choose an appropriate depth for your design element.

a) Create a formula for calculating the volume of material needed to fill your design element.

b) Use your formula to calculate the volume of material needed for widths of 2 m, 3 m, 4 m, and 5 m. Which width would you prefer for your design element? Why?

For example, a swimming pool may have a depth of 1.5 m.

Is a pool filled right to the top with water? How might this affect your formula?