7.1 Multiplying and Dividing Monomials

Focus on...

After this lesson, you will be able to...

- multiply a monomial by a monomial
- divide a monomial by a monomial

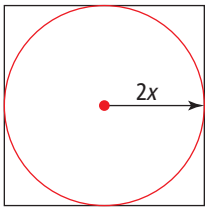
Did You Know?

The Medicine Wheel represents harmony and connection. It is an important symbol of the peaceful relationships among all living things.

Literacy Link

A *monomial* has one term. For example, 5, $2x$, $3s^2$, $-8cd$, and $\frac{n^4}{3}$ are all monomials.

The band council would like to design a Medicine Wheel similar to the one shown above for a square area of the new school courtyard. According to the design, the edges of the outer circular pathway will just touch the edges of the square. The outer radius of the circle can be represented by $2x$. How could you determine the relationship between the area of the circle and the area of the square?



Explore Multiplying and Dividing Monomials

1. What is the side length of the square in terms of x?

2. **a)** Write an expression for the area of the circle.
   b) Write an expression for the area of the square.

3. Show how to compare the two areas using a ratio expressed in lowest terms.

Do not use an approximate value for π. Leave π in the ratio.

4. How does the area of the square compare to the area of the circle?

Reflect and Check

5. Would this relationship be the same for any circle inscribed in a square? Explain.

6. **a)** How would you multiply the monomials $4x$ and $3x$?
   b) How would you divide the monomial $10x^2$ by the monomial $5x$?

Link the Ideas

Example 1: Multiply Monomials

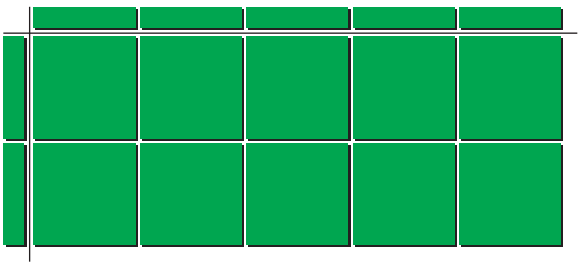
Determine each product.

a) $(5x)(2x)$ **b)** $(3x)(2y)$

Solution

a) *Method 1: Use a Model*

You can use x-tiles and x^2-tiles to model $(5x)(2x)$.



Each square has an area of $(x)(x) = x^2$. There are 10 positive x^2-tiles.
So, $(5x)(2x) = 10x^2$.

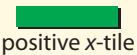
Method 2: Algebraically

Multiply the numerical coefficients.
Then, multiply the variables.

$(5x)(2x)$
$= (5)(2)(x)(x)$
$= 10x^2$

How can you use the exponent laws to help you multiply the variables?

You can use algebra tiles to model algebraic expressions.


positive x-tile


positive y-tile


positive xy-tile


positive x^2-tile

The same tiles in white represent negative quantities.

b) *Method 1: Use a Model*

You can use x-tiles, y-tiles, and xy-tiles to model $(3x)(2y)$.

Each grey rectangle has an area of $(x)(y) = xy$.
There are 6 positive xy-tiles. So, $(3x)(2y) = 6xy$.

Method 2: Algebraically

$(3x)(2y)$
$= (3)(2)(x)(y)$
$= 6xy$

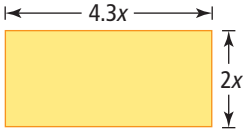
Show You Know

Determine each product in two different ways.

a) $(4x)(2y)$ **b)** $(-x)(7x)$

Example 2: Apply Monomial Multiplication

What is an expression for the area of the rectangle?



Solution

You can calculate the area, A, of a rectangle by multiplying the length by the width.

$A = (4.3x)(2x)$
$A = (4.3)(2)(x)(x)$
$A = (8.6)(x^2)$
$A = 8.6x^2$

An expression for the area of the rectangle is $8.6x^2$.

Show You Know

Calculate each product.

a) $(11a)(2b)$ **b)** $(-5x)(3.2)$

Example 3: Divide Monomials

Determine each quotient.

a) $\frac{-10x^2}{2x}$ **b)** $\frac{8xy}{4x}$

Solution

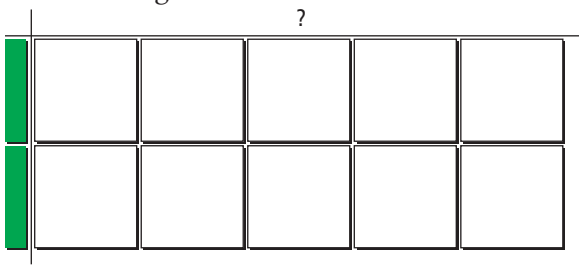
Strategies
Model It

a) *Method 1: Use a Model*

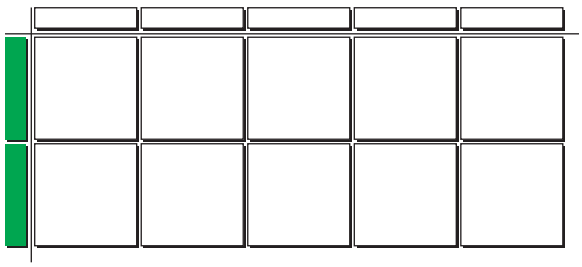
You can divide using algebra tiles.

Model $\frac{-10x^2}{2x}$ by representing the numerator with 10 negative x^2-tiles.

Arrange the 10 tiles into a rectangle so that one of the sides is 2 x-tiles long.



The unknown side length of the rectangle is made up of 5 negative x-tiles.



$\frac{-10x^2}{2x} = -5x$

Method 2: Algebraically

You can divide the numerator and the denominator by $2x$.

$$\frac{-10x^2}{2x}$$

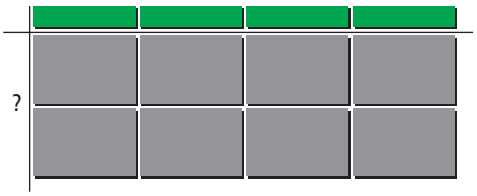
$$= \frac{\overset{-5x}{\cancel{-10x^2}}}{\underset{1}{\cancel{2x}}}$$

$$= -5x$$

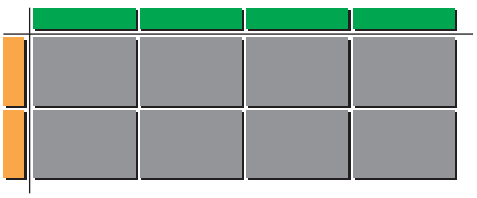
How can you use the exponent rules to help you divide the variables?

b) *Method 1: Use a Model*

Model $\frac{8xy}{4x}$ by representing the numerator with 8 xy-tiles. Arrange the 8 tiles into a rectangle so that one of the sides is 4 x-tiles long.



The unknown side length of the rectangle is made up of 2 y-tiles.



$$\frac{8xy}{4x} = 2y$$

Method 2: Algebraically

Divide common factors in the numerator and denominator.

$$\frac{8xy}{4x}$$

$$= \frac{\overset{2}{\cancel{8x}}y}{\underset{1}{\cancel{4x}}}$$

$$= 2y$$

Show You Know

Determine each quotient.

a) $\frac{12xy}{3y}$

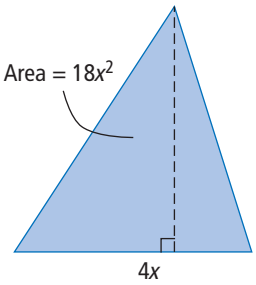
b) $\frac{-14x^2}{-2x}$

Example 4: Apply Monomial Division

The area of a triangle is given by the expression $18x^2$. The base of the triangle is represented by $4x$. What is the height of the triangle in terms of x?

Strategies
Draw a Diagram

Solution



The area of a triangle can be calculated by multiplying the base by the height, then dividing by 2.

$\text{Area} = \text{base} \times \text{height} \div 2$

So, if the area and base are known, then

$$\text{Height} = 2 \times \frac{\text{area}}{\text{base}}$$

$$\text{Height} = \frac{(2)(18x^2)}{4x}$$

$$\text{Height} = \frac{36x^2}{4x}$$

Divide the numerical coefficients.
Then, divide the variables.

$$\text{Height} = 9x$$

The height of the triangle is $9x$.

Show You Know

Calculate each quotient.

a) $\frac{18x^2}{3x}$

b) $14y \div (-2)$

c) $\frac{-18.6mn}{-3n}$

Key Ideas

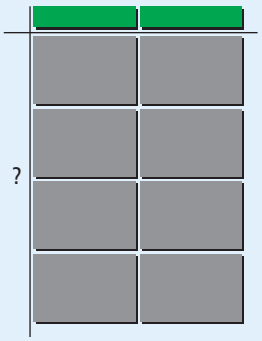
- You can represent the multiplication and division of monomials using a model.

$(2x)(-3x)$

There are 6 negative x^2-tiles.

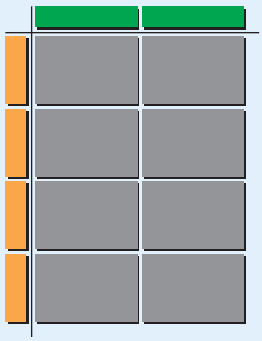
$(2x)(-3x) = -6x^2$

$\frac{8xy}{2x}$



The unknown side length of the rectangle is made up of 4 positive y-tiles.

$\frac{8xy}{2x} = 4y$



- To multiply monomials algebraically, you can multiply the numerical coefficients and use the exponent rules to multiply the variables.
- To divide monomials algebraically, you can divide the numerical coefficients and use the exponent rules to divide the variables.

Check Your Understanding

Communicate the Ideas

1. Explain to a partner at least two ways you could find the product of $(3x)$ and $(5x)$.

2. Laurie used the following method to divide $16n^2$ by $2n$.

$\frac{16n^2}{2n}$

$= (16 - 2)(n^2 - n)$

$= 14n$

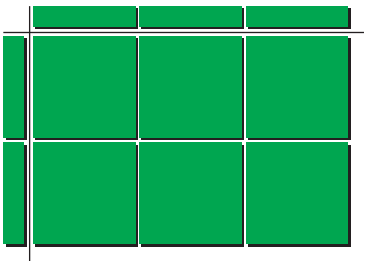
Does Laurie's method have any errors? If so, what are her errors and what is the correct solution?

Practise

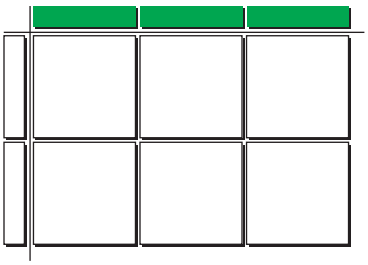
For help with #3 to #8, refer to Example 1 on page 255.

3. What multiplication statement is represented by each set of algebra tiles?

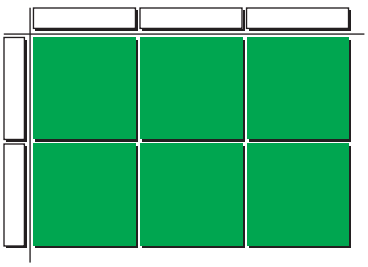
a)



b)

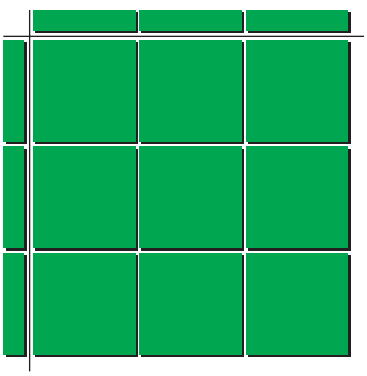


c)

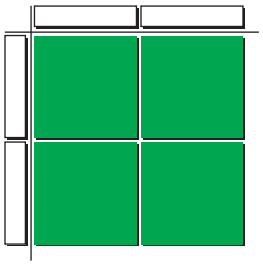


4. Determine the multiplication statement shown by each set of algebra tiles.

a)



b)



c)

5. Model and complete each multiplication statement.

a) $(2x)(4x)$ **b)** $(-4x)(2x)$

c) $(-4x)(-2x)$ **d)** $(-2x)(4x)$

6. Represent each multiplication statement with a model. Then, give the product.

a) $(3x)(5x)$ **b)** $(x)(-6x)$

c) $(-3x)(2x)$ **d)** $(-x)(x)$

For help with #7 to #10, refer to Example 2 on page 256.

7. Find the product of each pair of monomials.

a) $(2y)(5y)$ **b)** $(3a)(-6b)$

c) $(-q)(-9q)$ **d)** $\left(\frac{2}{3}x\right)(3x)$

e) $(-3r)(-2t)$ **f)** $(1.5p)(-3p)$

8. Multiply each pair of monomials.

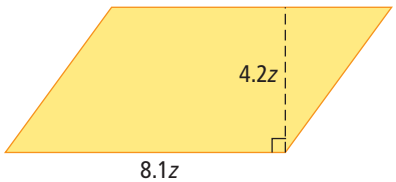
a) $(3n)(2n)$ **b)** $(-4k)(-7k)$

c) $(-4w)(2.5w)$ **d)** $\left(\frac{-3}{5}x\right)(15x)$

e) $(8m)(-0.5n)$ **f)** $(t)(-7t)$

9. A rectangle has a width of $3.9x$ and a length of $5x$. What is an expression for the area of the rectangle?

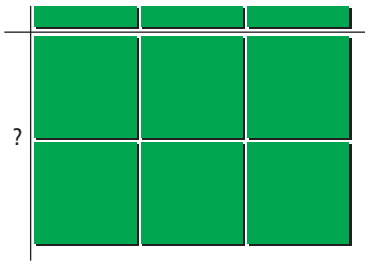
10. A parallelogram has a base of $8.1z$ and a height of $4.2z$. What is an expression for the area of the parallelogram?



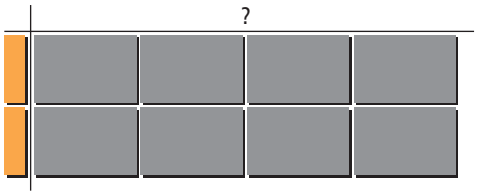
For help with #11 to #16, refer to Example 3 on pages 256-257.

11. Write the division statement represented by each set of algebra tiles.

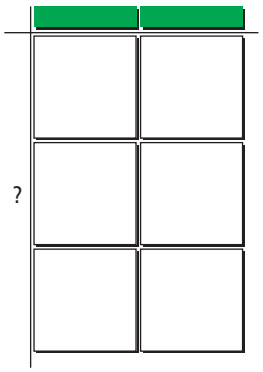
a)



b)

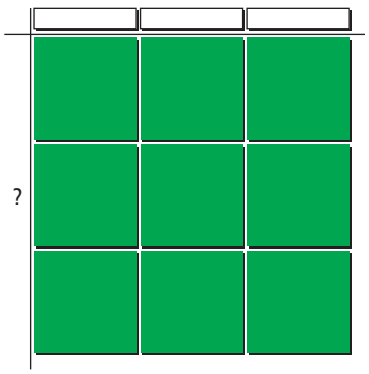


c)

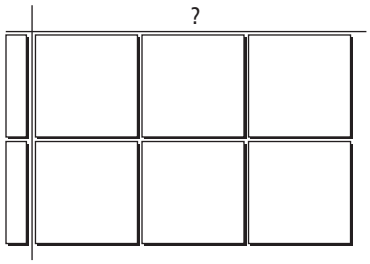


12. Determine the division statement shown by each set of algebra tiles.

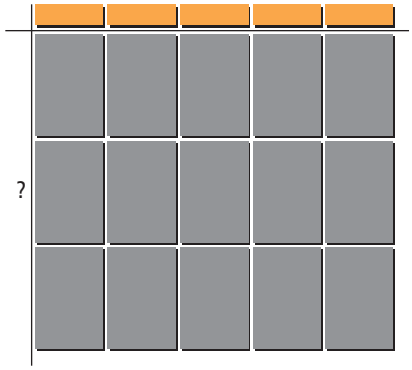
a)



b)



c)



13. Model and complete each division.

a) $\frac{8x^2}{2x}$ **b)** $\frac{5xy}{5y}$

c) $\frac{-12x^2}{4x}$ **d)** $\frac{2x^2}{-x}$

14. Model and complete each division.

a) $\frac{-15x^2}{3x}$ **b)** $\frac{10xy}{2x}$

c) $\frac{12x^2}{-3x}$ **d)** $\frac{-9x^2}{-3x}$

15. Find the quotient of each pair of monomials.

a) $\frac{7x^2}{x}$ **b)** $\frac{25st}{5s}$ **c)** $\frac{125t}{5}$

d) $\frac{-8m}{-2m}$ **e)** $\frac{81rs}{3rs}$ **f)** $\frac{4.5p^2}{-3p}$

16. Divide.

a) $12.4x^2 \div x$ **b)** $-15r \div (-4r)$

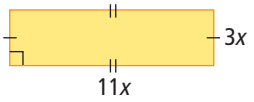
c) $0.6t^2 \div 0.2t^2$ **d)** $-18pn \div 3n$

e) $k \div 4k$ **f)** $\frac{2}{3}x^2 \div 2x$

Apply

17. Find an expression for the area of each figure.

a)



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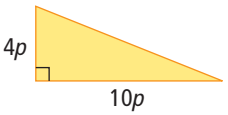
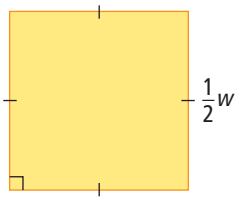
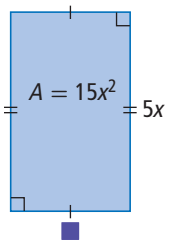


c)

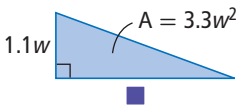


18. What is the missing dimension in each figure?

a)



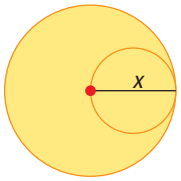
b)



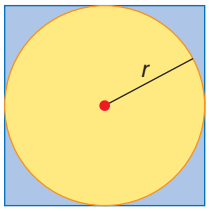
19. The area of a rectangle is $72d^2$ and its length is $20d$. What is an expression for its width?

20. Claire wants to build a patio outside her café. The rectangular space outside of Claire's café is three times as long as it is wide. The area of the space is 48 m^2. Claire would like to build a patio with dimensions 3.5 m by 12.5 m in this space. Will it fit? Explain.

21. The diagram shows that x is the radius of the large circle and the diameter of the small circle. Write the ratio of the area of the large circle to the area of the small circle. Simplify the expression.



22. A circle is inscribed in a square as shown.



In terms of the radius, r, determine each of the following ratios.

a) the area of the square to the area of the circle

b) the perimeter of the square to the circumference of the circle

Literacy Link

An *inscribed circle* fits exactly into another figure so that the edges of the two figures touch, but do not intersect.

23. Jonasie and Elisa are taking two tourists on a trip to photograph caribou. The visitors will be travelling by dogsled. The dogsled's length is 4 times its width. The sled has a rectangular base area of 3.2 m^2. The equipment to be loaded on the sled measures 0.8 m wide by 3.5 m long. Will the equipment fit on the sled as it is presently packed? Explain your answer.

Extend

24. A rectangular prism has a volume in cubic centimetres expressed as the monomial $60xy$. The length and width of the prism, in centimetres, are $4x$ and $3y$ respectively.

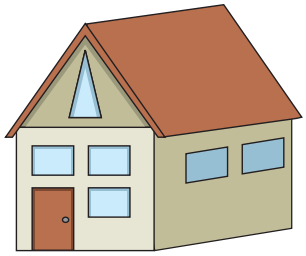
a) Determine the height.

b) Write an expression for the surface area of the rectangular prism.

25. How is determining $\frac{9n}{3n^2}$ similar to and different from determining $\frac{9n^2}{3n}$?

26. A and B represent monomials. When A is multiplied by B, the result is A. When A is divided by B, the result is A. What is the monomial B?

27. A contractor needs to order the glass for a window. The window is in the shape of an isosceles triangle and the height of the window is 2.5 times the base width.



a) Determine an expression for the area of the window in terms of the width of its base.

b) If the width of the base must be 85 cm, what is the largest window area the contractor can use?

Math Link

Landscape designs for gardens may include rectangular and circular areas for flower beds, lawns, patios, and pools. As a landscape designer, you sometimes need to:

- calculate the volume of material, such as soil, gravel, water, or mulch, needed to fill these areas to a certain depth
- calculate the area that a known volume of material will cover

The following are the formulas for these calculations:

Volume = area × depth

$$\text{Area} = \frac{\text{volume}}{\text{depth}}$$

- Draw a rectangle and circle that might be used in landscaping. Label the design element that each shape represents.
- Use variables for the dimensions of the shapes.
- Create an area formula for each shape.
- For each shape, tell what type of material you will use to fill it. Also, tell what the depth of the material will be.
- Create a volume formula for each shape.
- Along with each formula, include an explanation concerning any coefficients you use. For example, you may have to convert centimetre measurements to metres.

You may wish to create a spreadsheet that allows you to enter the values to calculate areas and volumes.

How do these coefficients relate to your landscape design?