6.3 Graphing Linear Relations

Focus on...

After this lesson, you will be able to...

- **graph linear relations**
- **match equations of linear relations with graphs**
- **solve problems by graphing a linear relation and analysing the graph**

Tina is in charge of ordering water supplies for a cruise ship. She knows the amount of water required per day for each passenger and crew member as well as the amount of water reserves that the ship carries. She decides to use her knowledge of linear relations to draw a graph representing the relationship between the amount of water needed and the length of a cruise.

If Tina were to develop an equation, how could she determine if the graph and the equation represent the same relationship?

Explore Graphs of Linear Relations

Materials

- grid paper
- ruler

On a cruise, the average person requires a minimum of 4 L of water per day. The cruise ship has capacity for 1500 passengers and crew. The ship also carries a reserve of 50 000 L of water in case of emergency.

1. a) Use a method of your choice to determine how much water will be needed each day of a seven-day cruise.

b) On grid paper, plot the data and label your graph. Compare your graph with that of a classmate.

What values will you plot along the horizontal axis? along the vertical axis?

2. a) Predict how much water is needed for a ten-day cruise.

b) What linear equation represents the litres of water needed per day?

c) How could you verify your answer for part a)? Try out your strategy.

Reflect and Check

3. Do your graph and the equation represent the same relationship? Explain.

4. Discuss with a partner if it would be appropriate to interpolate or extrapolate values using a fraction of a day. Explain why or why not.

5. a) If the cruise ship used 152 000 L of water, approximately how long did the trip last? Compare the method you used with a classmate's.

b) Is there more than one way to answer part a)? Explain. Which method seems more efficient?

Link the Ideas

Example 1: Graph a Linear Equation

The world's largest cruise ship, *Freedom of the Seas*, uses fuel at a rate of 12 800 kg/h. The fuel consumption, f, in kilograms, can be modelled using the equation $f = 12\,800t$, where t is the number of hours travelled.

a) Create a graph to represent the linear relation for the first 7 h.

b) Approximately how much fuel is used in 11 h? Verify your solution.

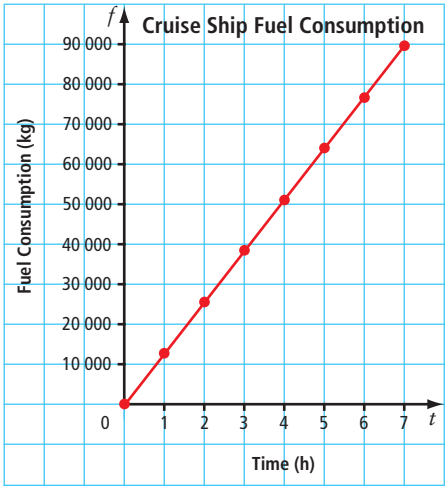
c) How long can the ship travel if it has approximately 122 000 kg of fuel? Verify your solution.

Solution

Method 1: Use Paper and Pencil

a) Create a table of values. Graph the coordinate pairs.

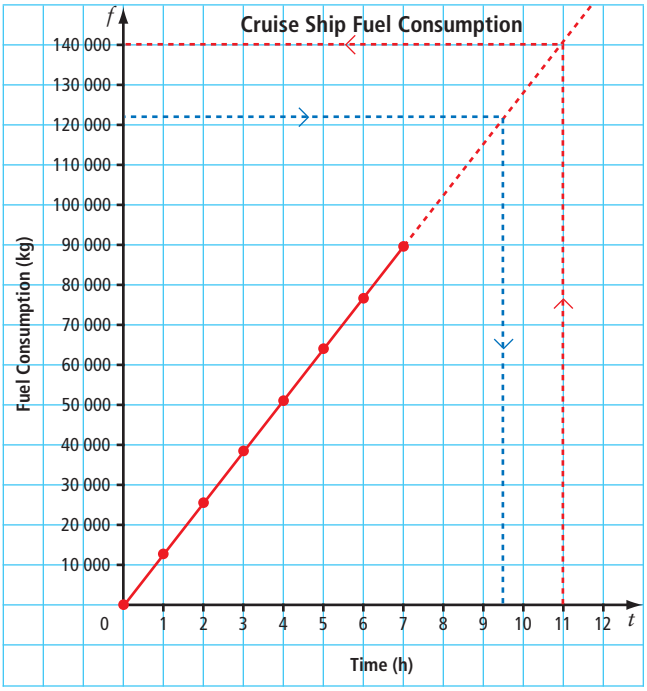
Time, t (h)	Fuel Consumption, f (kg)
0	0
1	12 800
2	25 600
3	38 400
4	51 200
5	64 000
6	76 800
7	89 600



Describe the connection between the equation and the graph.

b) Draw a straight line to connect the data points. Extend the line past the last data point.

Approximately 140 000 kg of fuel are used in 11 h.



What different methods might you use to represent and then solve the problem?

Check:
Substitute the value $t = 11$ into the equation $f = 12\,800t$.
$f = 12\,800(11)$
$= 140\,800$
The approximate solution is correct.

c) The fuel will last approximately 9.5 h.

Check:
Substitute $f = 122\,000$ into the equation and solve for t.
$122\,000 = 12\,800t$
$\frac{122\,000}{12\,800} = t$
$t \approx 9.53$
The approximate solution is correct.

Method 2: Use a Spreadsheet

a) In the spreadsheet, cell A1 has been labelled Time, t. Cell B1 has been labelled Fuel Consumption, f.

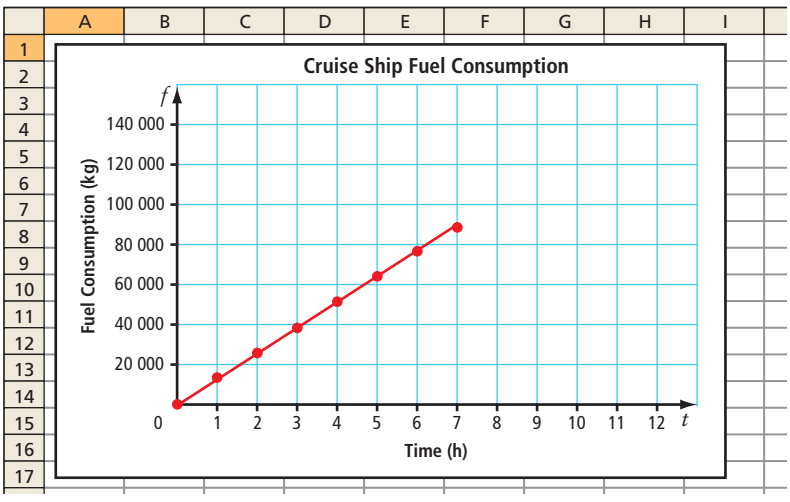
	A	B
1	Time, t(h)	Fuel Consumption, f(kg)
2	0	=12800*A2
3	1	12800
4	2	25600
5	3	38400
6	4	51200
7	5	64000
8	6	76800
9	7	89600

Enter the first eight values for t in cells A2 to A9. Then, enter the formula for the equation into cell B2. Use an = sign in the formula and * for multiplication. The value for t comes from cell A2.

Use the cursor to select cells B2 down to B9.

Then, use the **Fill Down** command to enter the formula in these cells. The appropriate cell for t will automatically be inserted. For example, =12800*A6 will be inserted into cell B6.

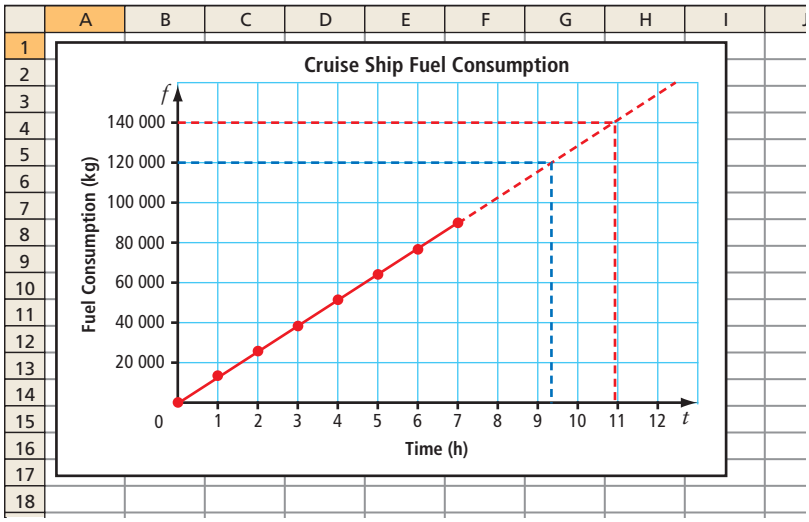
Use the spreadsheet's graphing command to graph the table of values. Note that different spreadsheets have different graphing commands. Use your spreadsheet's instructions to find the correct command.



Tech Link

You could use a graphing calculator to graph this linear relation. To learn about how to do this go to www.mathlinks9.ca and follow the links.

b) and **c)** From the menu, select **Add Trendline** to draw a straight line from the first data point to the last one. Extend the line past the last data point.



For part b), approximately 140 000 kg of fuel are used in 11 h.
For part c), the fuel will last approximately 9.5 h.

Did You Know?

Fish finders operate using sonar, which uses sound waves to "see" objects underwater. The fish finder produces a sound wave and sends it through the water. When the sound wave meets an object within its range, it bounces back to the fish finder. The fish finder determines the depth of the object by measuring the time between when the sound wave was sent and when it returns. The fish finder then sketches the object on the screen.

Show You Know

a) Graph the linear relation $y = 2x - 5$.

b) Use the graph to estimate the value of y if $x = 8$.

c) Use the graph to estimate the value of x if $y = -4$.

Example 2: Determine a Linear Equation From a Graph

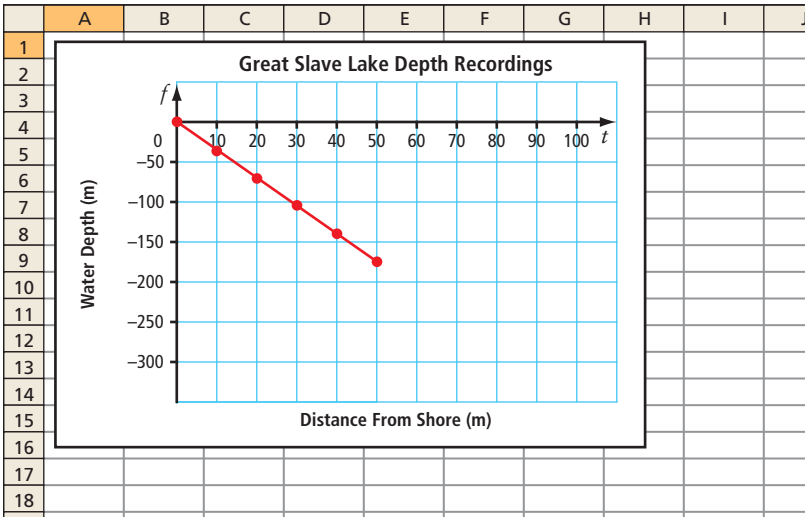
Great Slave Lake, which is located in the Northwest Territories, is the deepest lake in North America. It has a maximum depth of 614 m. Sam decided to check the depth using his fish finder. He collected the following data up to a depth of 180 m, which was the maximum depth that his fish finder could read.

Distance From Shore, d (m)	Water Depth, w (m)
0	0
10	−35
20	−70
30	−105
40	−140
50	−175

Literacy Link

A depth, such as 35 m, is expressed in different ways. In a table and a graph, use the negative value, −35. In a sentence, say "35 m below surface."

Sam used a spreadsheet to graph the data.



a) What linear equation does this graph represent? How do you know the equation matches the graph?
b) If this pattern continues, how far from shore would Sam be when the water is 614 m deep?
c) At what rate is the depth of the water decreasing?
d) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

Solution

a) Add a column to the table to help determine the pattern.

Distance From Shore, d (m)	Water Depth, w (m)	Pattern: Multiply d by −3.5
0	0	0
10	−35	−35
20	−70	−70
30	−105	−105
40	−140	−140
50	−175	−175

The water depth, w, decreases by 3.5 m for each 1-m increase in the distance from shore, d. The equation is $w = -3.5d$.

Check by substituting a known coordinate pair, such as (30, 105), into the equation.

Left Side $= -105$

Right Side $= -3.5(30)$
$= -105$

Left Side = Right Side

The equation is correct.

What is the connection between the graph and the equation?

Strategies

Solve an Equation

b) Substitute $w = 614$ into the equation and solve for d.

$$-614 = -3.5d$$

$$\frac{-614}{-3.5} = d$$

$$d \approx 175.4$$

Sam would be approximately 175.4 m from shore when the water is 614 m deep.

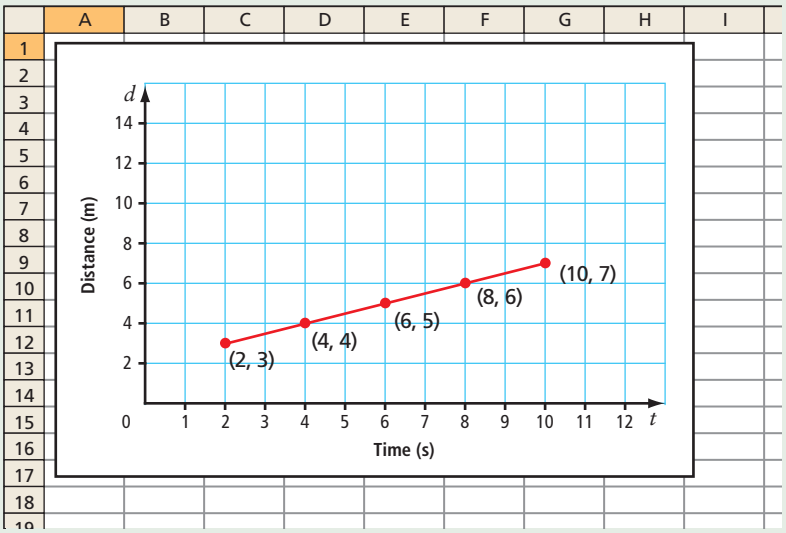
How else could you solve this problem?

c) The depth is decreasing at a rate of 3.5 m for each metre away from shore. The rate at which the water depth is decreasing is the coefficient of d in the equation.

d) Yes, it is reasonable to interpolate or extrapolate values between and beyond the given data points since the values for distance and depth exist. However, it is unreasonable to extrapolate values beyond the maximum depth of 614 m.

Show You Know

Identify the linear equation that represents the graph.



Example 3: Graph Horizontal and Vertical lines

For each table of values, answer the following questions:

Table 1

Time, t (s)	Distance, d (m)
0	6
30	6
60	6
90	6
120	6

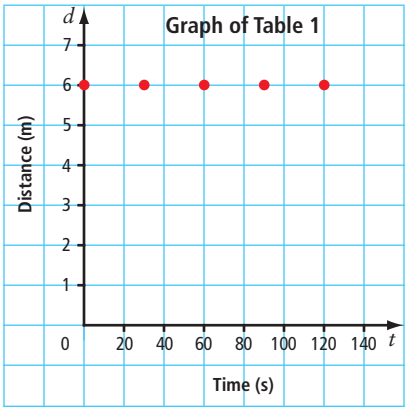
Table 2

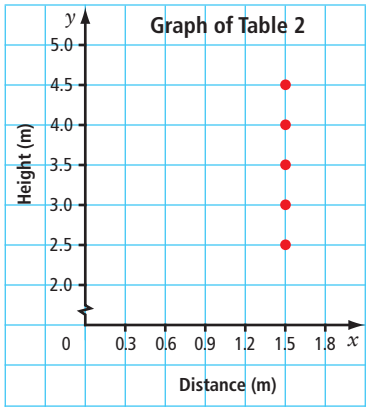
Distance, x (m)	Height, y (m)
1.5	2.5
1.5	3.0
1.5	3.5
1.5	4.0
1.5	4.5

a) Draw a graph to represent the table of values.
b) Describe a situation that the graph might represent.
c) Write the equation. Explain how you know the graph represents the equation.

Solution

a)





b) Table 1: The graph could show the relationship between distance and time when a pedestrian is waiting for a traffic light to change. The distance from the pedestrian to the opposite side of the road is constant.

Table 2: The graph could show the relationship between the height of a ladder and its distance from the wall where it is placed. The distance of the base of the ladder from the wall is constant as the ladder is extended.

Think of a different situation to represent each graph.

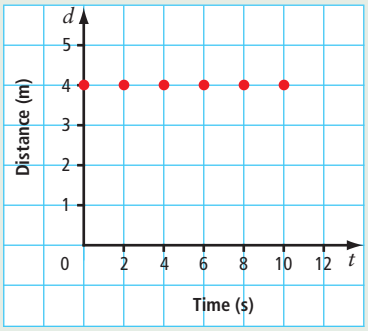
c) Table 1: The distance, d, remains constant for each interval of time.
The equation is $d = 6$.
For each value of t in the table and the graph, the value of d is 6.

Table 2: The distance, x, remains constant for each interval of height.
The equation is $x = 1.5$.
For each value of y in the table and the graph, the value of x is 1.5.

Show You Know

a) Write the linear equation that represents the graph.

b) Explain how you know the graph matches the equation.

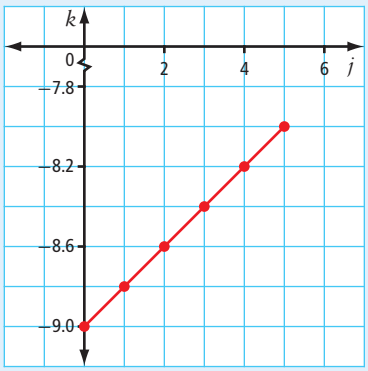


Key Ideas

- You can graph a linear relation represented by an equation.
  - Use the equation to make a table of values.
  - Graph using the coordinate pairs in the table. The graph of a linear relation forms a straight line.

$k = \frac{j}{5} - 9$

j	k
0	−9.0
1	−8.8
2	−8.6
3	−8.4
4	−8.2
5	−8.0



  - The graph of a linear relation can form a horizontal or a vertical line.
- You can use graphs to solve problems by interpolating or extrapolating values.

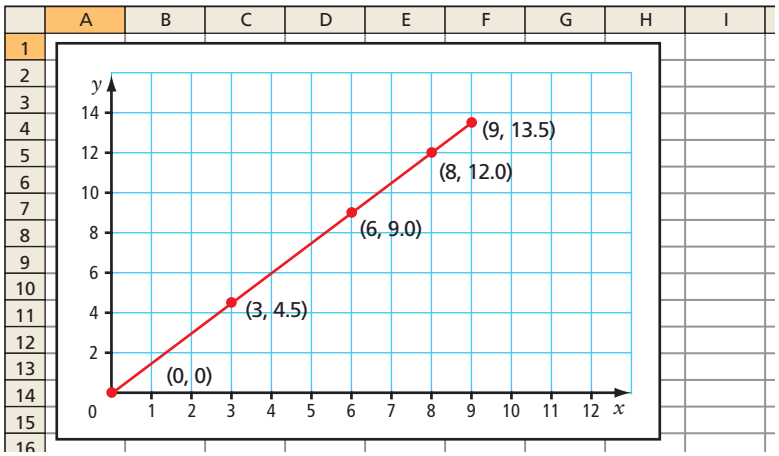
Check Your Understanding

Communicate the Ideas

1. You are given a linear equation. Describe the process you would follow to represent the equation on a graph. Use an example to support your answer.

2. Use examples and diagrams to help explain how horizontal and vertical lines and their equations are similar and how they are different.

3. a) Describe a real-life situation to represent the data on this graph.



b) Explain how you would determine the equation that represents the graph. Give your explanation to a classmate.

c) Can you interpolate or extrapolate values on this graph? Explain your thinking.

WWW Web Link

For practice matching graphs and linear equations, go to www.mathlinks9.ca and follow the links.

Practise

For help with #4 to #7, refer to Example 1 on pages 232–234.

4. Ian works part-time at a movie theatre. He earns $8.25/h. The relationship between his pay, p, and the time he works, t, can be modelled with the equation $p = 8.25t$.

a) Show the relationship on a graph.

b) Explain how you know the graph represents the equation.

c) Ian works 8 h in one week. Use two methods to determine his pay.

5. Andrea is travelling by bus at an average speed of 85 km/h. The equation relating distance, d, and time, t, is $d = 85t$.

a) Show the relationship on a graph.

b) How long does it take Andrea to travel 300 km?

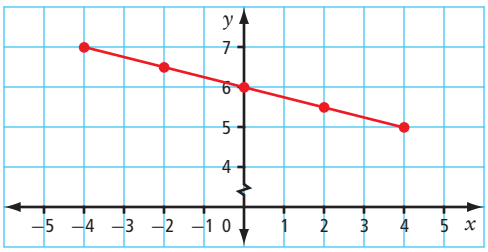
6. Choose the letter representing the graph that matches each linear equation.

a) $y = 5x$

b) $y = -2x + 3$

c) $y = -\frac{x}{4} + 6$

A



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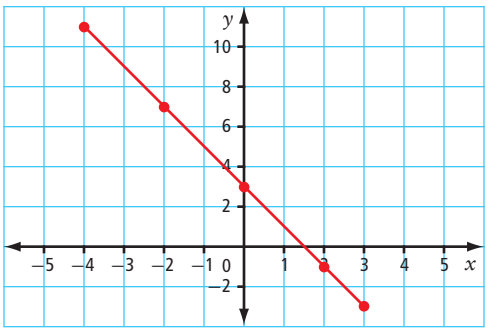
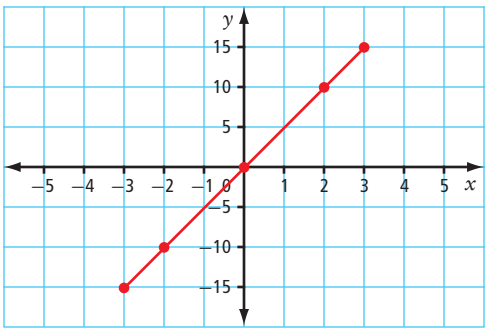


C



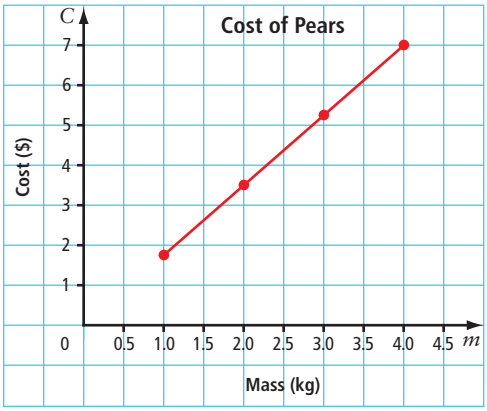
7. Create a table of values and a graph for each linear equation.

a) $x = 4$ **b)** $r = -3s + 4.5$

c) $m = \frac{k}{5} + 13$

For help with #8 to #11, refer to Example 2 on pages 234–236.

8. The graph shows the relationship between the cost, C, in dollars and the mass, m, in kilograms of pears.

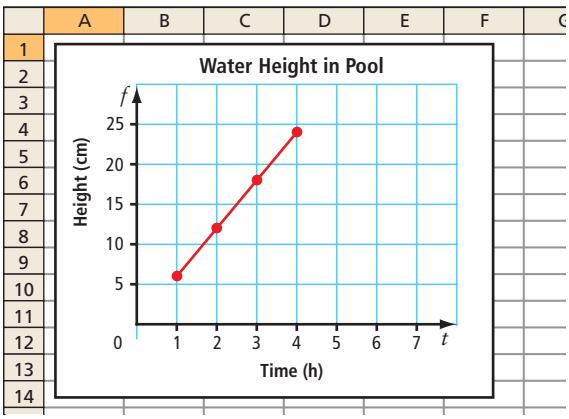


a) What is the linear equation?

b) How much could you buy for $5?

c) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

9. The graph represents the relationship between the height of water in a child's pool, h, and the time, t, in hours as the pool fills.



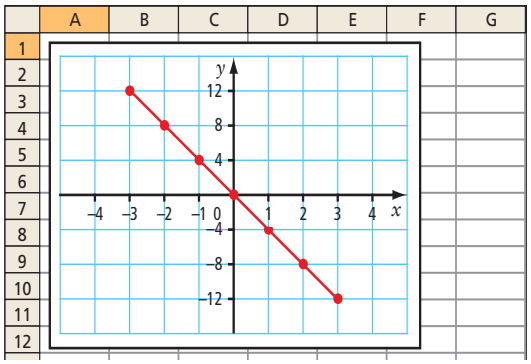
a) Determine the linear equation.

b) What is the height of the water after 5 h?

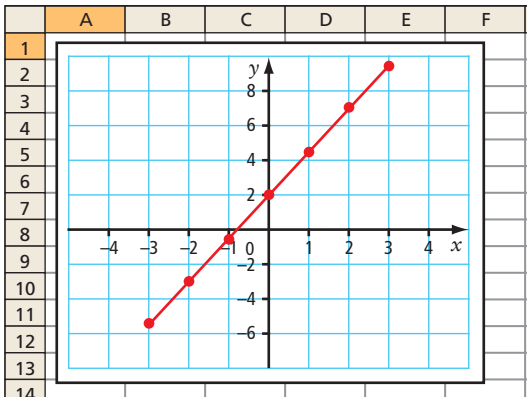
c) Is it appropriate to interpolate or extrapolate values on this graph? Explain.

10. Determine the linear equation that models each graph.

a)

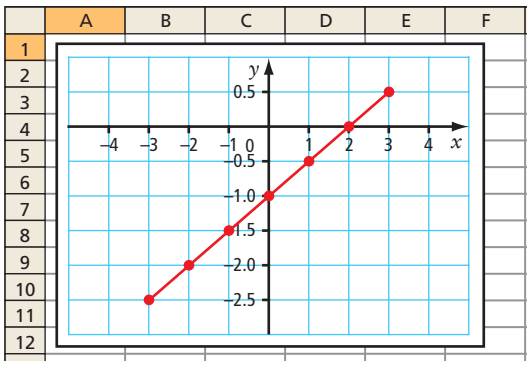


b)

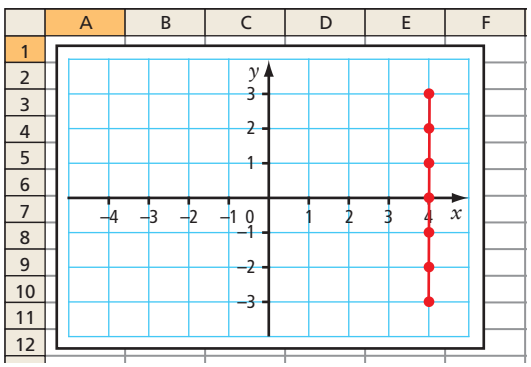


11. What linear equation does each graph represent?

a)



b)



12. Create a graph and a linear equation to represent each table of values.

a)

x	y
−3	−10
−2	−7
−1	−4
0	−1
1	2
2	5
3	8

b)

r	t
−3	−2.5
−2	−1.0
−1	0.5
0	2.0
1	3.5
2	5.0
3	6.5

c)

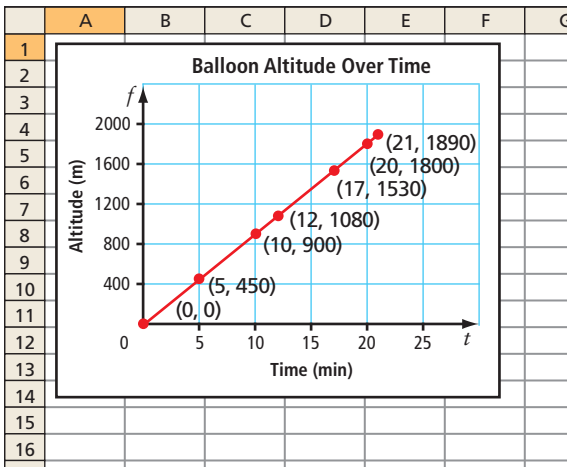
f	z
−3	−3
−2	−3
−1	−3
0	−3
1	−3
2	−3
3	−3

d)

h	n
−3	−0.75
−2	−0.5
−1	−0.25
0	0
1	0.25
2	0.5
3	0.75

Apply

13. The graph represents the altitude of a hot-air balloon the first 20 min after it was released.



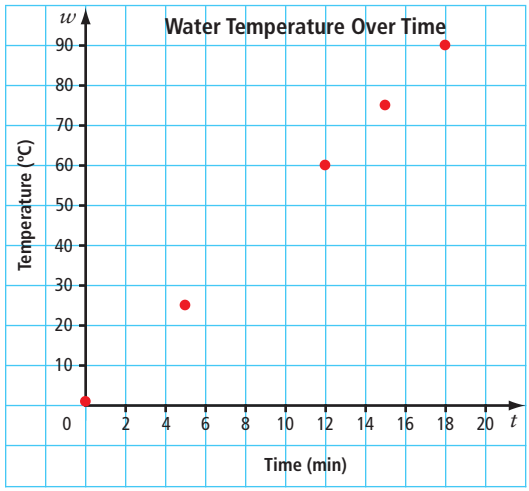
a) What was the approximate altitude of the balloon after 15 min?

b) Estimate how long it took for the balloon to rise to an altitude of 1 km.

c) What linear equation models the graph?

d) How fast is the balloon rising?

14. Sanjay conducted an experiment to determine how long it takes to heat water from 1 °C to its boiling point at 100 °C. He plotted his data on a graph.



a) Approximately how long did it take for the water to reach boiling point? Explain your reasoning.

b) What was the temperature of the water after 10 min?

c) At what rate did the water temperature increase? Explain your reasoning.

15. Paul drives from Edmonton to Calgary. He uses a table to record the data.

Time, t (h)	Distance, d (km)
0.5	55.0
0.9	99.0
1.2	132.0
1.5	165.0
2.3	253.0
2.7	297.0

a) Graph the linear relation.

b) How far did Paul drive in the first 2 h?

c) How long did it take Paul to drive 200 km?

d) Write the equation that relates time and distance.

e) What was Paul's average driving speed? What assumptions did you make?

WWW Web Link

To learn about using a graphing calculator to enter data on a table and plot the data on a graph, go to www.mathlinks9.ca and follow the links.

16. The relationship between degrees Celsius (°C) and degrees Fahrenheit (°F) is modelled by the equation $F = \frac{9}{5}C + 32$.

a) Graph the relationship for values between −50 °C and 120 °C.

b) Water boils at 100 °C. What is this temperature in degrees Fahrenheit?

c) Water freezes at 0 °C. How did you represent this on your graph?

d) At what temperature are the values for °C and °F the same?

17. Scuba divers experience an increase in pressure as they descend. The relationship between pressure and depth can be modelled with the equation $P = 10.13d + 102.4$, where P is the pressure, in kilopascals, and d is the depth below the water surface, in metres.

a) Graph the relationship for the first 50 m of diving depth.

b) What is the approximate pressure at a depth of 15 m? Verify your answer.

c) The maximum pressure a scuba diver should experience is about 500 kPa. At what depth does this occur? Verify your answer.

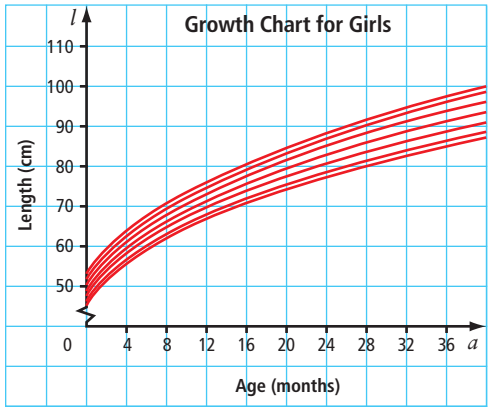
d) What does "+ 102.4" represent in the equation? How is it represented on the graph?

Did You Know?

After deep or long dives, scuba divers need to undergo decompression. They do this by ascending to the surface slowly in order to avoid decompression sickness, also known as the bends.

Extend

18. The graph shows the normal range of length for girls from birth to age 36 months.



a) For what age range does girls' growth appear to represent a linear relation?

b) For what age range, does girls' growth appear to represent a non-linear relation?

19. Janice left the school at 12 noon riding her bike at 20 km/h. Flora left school at 12:30 riding her bike at 24 km/h.

a) Draw a distance–time graph to plot the data for both cyclists during the first four hours. Use a different colour for each cyclist.

b) How can you tell from the graph that Flora has caught up to Janice?

c) About what time did Flora catch up to Janice?

d) If Janice and Flora continued to ride at their respective speeds, at what time would they again be apart by a distance of 2 km?

20. An online music download site offers two monthly plans. Plan A offers $10 plus $1 per download and Plan B offers $1.50 per download.

a) Graph both linear relations on the same grid.

b) Explain the conditions under which each deal is better.

21. Simple interest is paid according to the formula $I = p \times r \times t$, where p is the principal, r is the rate of interest per year, and t is the time in years. The interest is not added to the principal until the end of the time period. Canada Savings Bonds offer a simple interest bond payable at 3.5% per year up to a maximum of ten years.

a) Create a table of values to show the interest earned on a $1000 bond for the ten-year period.

b) Use a graph to show the interest earned over ten years.

c) How many years would it take to earn $100 interest? $200 interest?

d) If you could leave the principal beyond the ten-year period, estimate the number of years it would take to earn $500 interest.

Math Link

The world's fastest submarines can reach speeds of 74 km/h in 60 s, starting from rest. If a submarine is already moving, then the time to reach its top speed will differ.

a) Choose four different starting speeds up to a maximum of 74 km/h. For each speed, assume that the acceleration is the same. For each speed include:
- a table of values
- a linear equation and a graph to represent the relationship between speed and time

b) Describe each graph. Identify any similarities and differences you observe between the graphs and the equations.

Did You Know?

A student team from the University of Québec set a new world speed record for the fastest one-person, non-propeller submarine. In 2007, the submarine, *OMER 6*, reached a speed of 4.642 knots (8.6 km/h) in the International Submarine Races.