## 6.2

## Interpreting Graphs

## Focus on...

After this lesson, you will be able to...

- describe patterns found in graphs
- extend graphs to determine an unknown value
- estimate values between known values on a graph
- estimate values beyond known values on a graph


Richard is paid a daily salary and commission to sell riverboat cruises.
As a salesperson, he recognizes that it is important to know his sales in order to determine his commission. He is considering using a graph to calculate this amount. What do the variables $p$ and $s$ represent? What other information does the graph provide?

## Explore Using a Graph to Solve Problems

Richard is paid $\$ 64 /$ day plus a commission of $10 \%$ of sales for selling riverboat cruises.

1. The table provides some information about his sales and daily pay.

| Day | River Cruises Sold (\$) | Pay for the Day (\$) |
| :--- | :---: | :---: |
| Monday | 1500 | 214 |
| Tuesday | 1250 |  |
| Wednesday | 800 |  |
| Thursday | 0 | 64 |
| Friday | 2010 |  |

a) On the graph, locate the data points for each day.
b) How could you use the graph to estimate the missing values on the table? Try your method.
2. a) Estimate how much Richard must sell to earn $\$ 300$ in one day. Describe your strategy.
b) Estimate how much Richard would earn if he had sales of $\$ 1150$. Describe your strategy.
3. The graph represents the linear equation $p=\frac{s}{10}+64$. How could you use this information to determine the answers to \#2? Explain why your strategy is effective.

## Reflect and Check

4. a) Explain how your graph helped you to answer \#2.
b) Discuss your strategies with a classmate.
5. Work with a partner and use your graph.
a) How much would Richard earn if he had sales of $\$ 2400$ ?
b) How much must Richard sell to earn $\$ 175$ in one day?
6. a) List an advantage and a disadvantage of using a graph and an equation to determine values.
b) Which method is more effective when

- estimating pay given the sales?
- determining sales given the pay?


## Link the Ideas

## interpolate

- estimate a value between two given values
- interpolation should be used only when it makes sense to have values between given values. For example, 5.4 people does not make sense.


## Literacy Link

 When describing variables on a graph, express the $y$-variable in terms of the $x$-variable. For example, the graph shows the relationship between temperature and altitude.
## Tech Link

You can use a spreadsheet program to create the graph.

## Strategies

Estimate and Check

## Example 1: Solve a Problem Using Interpolation

A weather balloon recorded the air temperature at different altitudes. The data approximately represent a linear relationship.

| Altitude, $\boldsymbol{a}(\mathbf{m})$ | 350 | 750 | 1000 | 1500 | 1800 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Temperature, $\boldsymbol{t}\left({ }^{\circ} \mathbf{C}\right)$ | 11.4 | 5.7 | 2.1 | -5.0 | -10.0 |

a) Interpolate the approximate value for the air temperature when the balloon was at a height of 600 m .
b) What was the approximate altitude of the balloon at an air temperature of $-7.5^{\circ} \mathrm{C}$ ?
c) Is it possible to interpolate the precise value for the air temperature when the altitude is 1050.92 m ? Explain.

## Solution

Graph the data. Since temperature change is continuous, you can draw a straight line to connect the data points.

a) On the graph, draw a vertical line from 600 to the point where the line intersects the graph. From the intersection point, draw a horizontal line to the $y$-axis. The value where the horizontal line meets the $y$-axis is approximately 8 .
b) On the graph, draw a horizontal line from -7.5 to the point where the line intersects the graph. From the intersection point, draw a vertical line to the $x$-axis. The value where the vertical line meets the $x$-axis is approximately 1700 .
c) No, you cannot interpolate a precise value. The temperature is related to altitude values to the nearest 50 m . There is too much uncertainty to accurately predict temperatures based on altitude measurements that have a precision of $\frac{1}{100}$ of a metre. You could, however, estimate the temperature for an altitude of between 1000 m and 1100 m .

## Show You Know

This graph shows a plane's altitude as it lands. The relationship between altitude and time is approximately linear.
a) What was the plane's approximate altitude at 50 s ?
b) At what time was the plane's altitude approximately 11 km ?
c) Is it appropriate to join the points with
 a straight line? Explain.

## Example 2: Solve a Problem Using Extrapolation

Anna is kayaking up the east coast of the Queen Charlotte Islands toward Graham Island.

Anna's course is shown by the red arrow on the map.
a) If Anna continues on her present course, extrapolate the values of the coordinates for latitude and longitude where she will land.
b) Could you use extrapolation to estimate where Anna sailed from? Explain.


## extrapolate

- estimate a value beyond a given set of values
- extrapolation should be used only when it makes sense to have values beyond given values


## (6) Did You Know?

The Queen Charlotte Islands or Haida Gwaii consist of two main islands off the northwest coast of British Columbia. In addition to Graham Island and Moresby Island, there are approximately 150 smaller islands.

## Solution

a) Extend the line that represents the course until it touches land, and then read the coordinates of the point. From the map, the coordinates where Anna will land are approximately $53.8^{\circ}$ north latitude and $-131.8^{\circ}$ west longitude.
b) No, the line extended in the direction that Anna sailed from does not strike land. This is evidence that this extended line does not show the place where she started.


## CD Literacy Link



Latitude and longitude is the most common grid system used for navigation.

- Each degree of latitude is approximately 111 km apart.
- Each degree of longitude varies from 0 to 111 km . A degree of longitude is widest at the equator and gradually decreases at the poles.
An example of a coordinate reading from a Canadian location might be $61^{\circ}$ north latitude and $-139^{\circ}$ west longitude.


## Show You Know

The value of a computer decreases over time. The graph shows the value of a computer after it was bought.
a) After what approximate period of time does the computer have no value?
b) When was the computer worth approximately $\$ 200$ ?
c) Is it appropriate to join the points with a straight line? Explain.

## Key Ideas

- On a graph, you can use a line to interpolate values between known values.
- Start with a known value for $x$.

- Start with a known value for $y$.

- On a graph, you can extend a line to extrapolate values beyond known values.
- Use a dashed line to extend the line beyond the known $x$-value or $y$-value.
- Start with a known value for $x$.

- Start with a known value for $y$.

- Interpolation and extrapolation should be used only when it is reasonable to have values between or beyond the values on a graph.


## Check Your Understanding

## Communicate the Ideas

1. Josh asked you to help him understand interpolation and extrapolation. Use an example and a graph to help explain how interpolation and extrapolation are similar and how they are different.
2. Grace says it would be reasonable to interpolate values on these graphs. Is she correct? Explain.


3. Develop a situation that involves a linear relation. Draw and label the corresponding graph. Develop a question and answer that requires extrapolating a value on the graph. Compare your work with that of a classmate.

## Practise

For help with \#4 to \#7, refer to Example 1 on page 222.
4. The graph shows a linear relation between distance and time.

a) What is the approximate value of the $d$-coordinate when $t=5$ ? Explain the method you used to determine the answer.
b) What is the approximate value of the $t$-coordinate when $d=20$ ?
5. The graph shows a linear relation.

a) What is the approximate value of the $y$-coordinate when $x=-2.5$ ?
b) What is the approximate value of the $x$-coordinate when $y=4$ ?
6. The graph shows a linear relation.

a) What is the approximate value of the $y$-coordinate when $x=1.5$ ?
b) What is the approximate value of the $x$-coordinate when $y=0.5$ ?
7. a) The table of values represents the distance that Sophie cycles in relation to time.

| Time, $\boldsymbol{t}$ (h) | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance, $\boldsymbol{d}$ <br> $(\mathbf{k m})$ | 12.5 | 25 | 37.5 | 50 | 62.5 | 75 |

a) Plot the linear relation on a graph.
b) From the graph, determine the approximate distance Sophie has cycled after 2.5 h .
c) From the graph, approximate how long it takes Sophie to cycle 44 km .

## For help with \#8 to \#11, refer to Example 2 on

 pages 223-224.8. The graph shows a linear relation between distance and time.

a) What is the approximate value of the $d$-coordinate when $t=10$ ?
b) What is the approximate value of the $t$-coordinate when $d=33$ ?
9. The graph shows a linear relation.

a) What is the $y$-coordinate when $x=-3$ ?
b) What is the $x$-coordinate when $y=1$ ?
10. The graph shows a linear relation.

a) What is the approximate value of the $y$-coordinate when $x=-4$ ?
b) What is the approximate value of the $x$-coordinate when $y=-6$ ?
11. The table of values represents the drop in temperature after noon on a winter day.

| Time, $\boldsymbol{t}(\mathbf{h})$ | $1: 00$ | $2: 00$ | $3: 00$ | $4: 00$ | $5: 00$ | $6: 00$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature, <br> $\boldsymbol{d}\left({ }^{\circ} \mathbf{C}\right)$ | 1 | 0 | -1 | -2 | -3 | -4 |

a) Plot the data on a graph.
b) From the graph, what is the approximate temperature at 6:30?
c) From the graph, determine the approximate time when the temperature is $2^{\circ} \mathrm{C}$.

## Apply

12. a) In a bulk food store, trail mix costs $\$ 2.40$ per 250 g . Plot the data on a graph.

| Mass of <br> Trail Mix, <br> $\boldsymbol{m}(\mathbf{g})$ | 250 | 500 | 750 | 1000 | 1250 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Cost, $\mathbf{C}$ (\$) | 2.40 | 4.80 | 7.20 | 9.60 | 12.00 |

b) From the graph, approximate how much 2000 g of trail mix would cost.
c) From the graph, approximate how much trail mix you would get for $\$ 13$.
13. The submarine HMCS Victoria can dive to a depth of 200 m .


a) Is it reasonable to interpolate or extrapolate values on this graph? Explain.
b) How long does it take Victoria to reach a depth of 140 m ?
c) What is the submarine's depth after 4 min?
14. A grade 9 class earns a profit of $53 \notin$ for each program they sell for the school play.

a) Is it reasonable to interpolate or extrapolate values on this graph? Explain.
b) If 500 programs were sold, how much profit would the students make? What strategy did you use to solve the problem?
c) Approximately how many programs would students need to sell in order to earn $\$ 2500$ ?
15. Sean learned in his cooking class that the time it takes to cook a roast depends upon its mass. The graph shows the relationship between cooking time and the mass of a roast.


From the graph, determine the approximate cooking time for a roast with each given mass.
a) 1.25 kg
b) 2.25 kg
c) 4.2 kg
16. A cell phone company charges a $\$ 33.95$ monthly fee and long-distance charges at a rate of $\$ 0.35$ per minute. The graph shows the monthly cost of phone calls based on the number of long-distance minutes.

a) Is it reasonable to interpolate or extrapolate values on this graph? Explain.
b) What would be the approximate monthly bill for 60 min of long-distance calls?
c) Approximately how many minutes of long-distance calls could you buy for $\$ 50$ ?
17. The graph represents the relationship between the cost of renting a power washer and rental time.

a) How much does it cost to rent a power washer for four days? What is the cost per day? How do you know?
b) How long could you rent the power washer if you had $\$ 420$ ?

## Extend

18. The table shows the relationship between stopping distance and speed of a vehicle.

| Speed, $\mathbf{s}$ <br> $(\mathbf{k m} / \mathbf{h})$ | 15 | 30 | 45 | 60 | 75 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Stopping <br> Distance, $\boldsymbol{d}(\mathbf{m})$ | 6 | 15 | 28 | 42 | 65 |

a) Plot the data on a graph. Draw a line to join the data points to best approximate the trend.
b) What happens to stopping distance as speed increases?
c) Estimate the stopping distances for speeds of $5 \mathrm{~km} / \mathrm{h}, 55 \mathrm{~km} / \mathrm{h}$, and $80 \mathrm{~km} / \mathrm{h}$.
d) Estimate the speed before a driver applied the brake for stopping distances of 10 m , 50 m , and 100 m .
e) About how much farther is the stopping distance at $50 \mathrm{~km} / \mathrm{h}$ than it is at $30 \mathrm{~km} / \mathrm{h}$ ? at $70 \mathrm{~km} / \mathrm{h}$ than at $50 \mathrm{~km} / \mathrm{h}$ ?
f) Why do you think the graph is not a straight line?

19. The speed of a falling skydiver is shown for the first 3 s .

a) Approximately how long would it take for the skydiver to reach terminal velocity?
b) Approximately how far would the skydiver fall in that time?
c) Why do you think the graph is approximately a straight line?

## C. Did You Know?

Terminal velocity is the maximum speed that a skydiver can reach when falling. Air resistance prevents most skydivers in free fall from falling any faster than $54 \mathrm{~m} / \mathrm{s}$.

## Math Link

The area of the ocean called the Intertropical Convergence Zone (ITCZ) has little or no wind. Before propellers and motors, sailors used a relatively light anchor called a kedge to help them move across this region. The kedge anchor, which was attached to a line, was rowed out approximately 650 m ahead of the ship and dropped to the sea floor. A crew on the ship then grabbed the line and hauled it in to pull the ship to the anchor, a distance of 650 m . This process, called kedging, was repeated until the boat passed through the ITCZ.
a) Create a table of values showing the relationship between the number of kedges and the total distance travelled.
b) Plot the data on a graph. Label the graph.
c) Determine the value for how many kedges it would take to traverse the width of the ITCZ.
d) How did the skills you have learned in this chapter help you solve part c)?

## WWW Web Link

To learn more about the Intertropical Convergence Zone (ITCZ), go to www. mathlinks9.ca and follow the links.


## Did You Know?

The ITCZ is located between $5^{\circ}$ north and $5^{\circ}$ south of the equator and is approximately 1100 km wide. Note how the position of the ITCZ moves during the year.


