

# 5.3

## Adding and Subtracting Polynomials

### Focus on...

After this lesson, you will be able to...

- add polynomial expressions
- subtract polynomial expressions
- solve problems using the addition and subtraction of polynomials

A music store rents out a drum kit for \$55 per month, plus a deposit of \$30. Is there a pattern? How could you use a polynomial expression to represent this pattern?



### Explore Adding or Subtracting Polynomial Expressions

1. In first semester, Kira decides to play drums for music class. To rent a drum kit, it costs \$55 per month, plus a \$30 deposit.
  - a) What is the total cost of renting the drum kit for three months, including the deposit?
  - b) Write an expression to show the total cost for any number of months, including the deposit. Tell what your variable represents.
2. In second semester, Kira decides to play electric guitar. To rent an electric guitar, it costs \$22 per month, plus a \$20 deposit.
  - a) What is the total cost of renting an electric guitar for three months, including the deposit?
  - b) Write an expression to show the total cost for any number of months, including the deposit.
3. Mark wants to learn to play both drums and electric guitar. What is an expression for the total cost of renting a drum kit and a guitar for any number of months, including deposits? Then, show how to find a simpler expression.
4. What is an expression for the difference between the cost of renting a drum kit and the cost of renting a guitar for any number of months, including deposits? Then, show how to find a simpler expression.

Use the same variable that you used in #1. Why might you be able to use the same variable?

### Reflect and Check

5.
  - a) Describe how to add or subtract polynomial expressions.
  - b) Why was it necessary to use the same variable for each expression?
6. Make up your own situation that involves the rental of two items. Write an expression for the total cost and for the difference in cost.

## Link the Ideas

### Example 1: Add Polynomials

Add  $3x - 4$  and  $2x + 5$ . Simplify your answer by combining like terms.

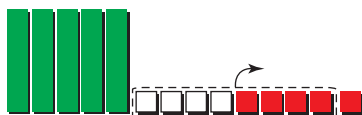
#### Solution

##### Method 1: Use a Model

You can use algebra tiles to model each polynomial.



Arrange the model so that like objects are together.  
Remove zero pairs if necessary.



Rewrite the model in simplest form.



$$(3x - 4) + (2x + 5) = 5x + 1$$

##### Method 2: Use Symbols

You can combine like terms.

$$\begin{aligned} &(3x - 4) + (2x + 5) \\ &= 3x + 2x - 4 + 5 \\ &= 5x + 1 \end{aligned}$$

What is another way to model the polynomials?

#### Strategies

#### Model It

### Show You Know

Use two methods to show each addition of polynomials. Give your answers in simplest form.

a)  $(2a - 1) + (6 - 4a)$

b)  $(3t^2 - 5t) + (t^2 + 2t + 1)$



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**Strategies**  
Model It

**Example 2: Determine Opposite Expressions**

What is the opposite for each of the following?

- a)  $3x$
- b)  $-2$
- c)  $4x - 1$
- d)  $a^2 - 3a + 2$

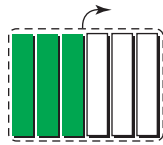
**Solution**

- a) You can use algebra tiles to represent  $3x$ .

How might you use a diagram to model  $3x$ ?



Add three negative  $x$ -tiles to give zero.



The opposite of  $3x$  is  $-3x$ .

You can also use inspection.



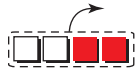
$3x$  is positive. The opposite of positive is negative.

The opposite of  $3x$  is  $-3x$ .

- b) You can use algebra tiles to represent  $-2$ .



Add two positive  $1$ -tiles to give zero.



The opposite of  $-2$  is  $+2$  or  $2$ .

You can also use inspection.



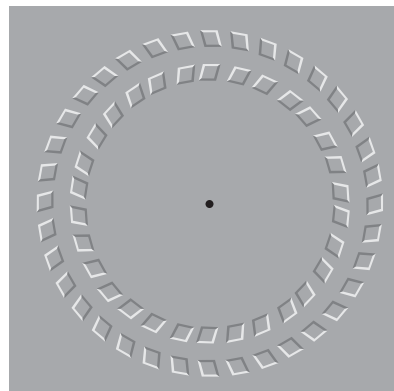
$-2$  is negative. The opposite of negative is positive.

The opposite of  $-2$  is  $+2$  or  $2$ .

**Art Link**

Opposites can be used to create interesting optical illusions.

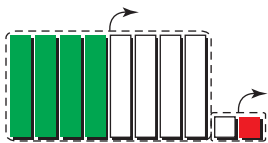
Look at the dot in the center of the rings. Lean forward and backward. The rings appear to rotate in opposite directions.



- c) You can use algebra tiles to represent  $4x - 1$ .



Add four negative  $x$ -tiles and one positive 1-tile to give zero.



The opposite of  $4x - 1$  is  $-4x + 1$ .

You can also use inspection.  M E

$4x$  is positive. The opposite of positive is negative.

The opposite of  $4x$  is  $-4x$ .

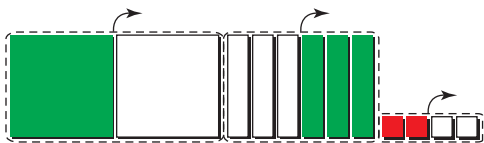
1 is being subtracted. The opposite of subtracting 1 is adding 1.

The opposite of  $4x - 1$  is  $-4x + 1$ .

- d) You can use algebra tiles to represent  $a^2 - 3a + 2$ .



Add a negative  $a^2$ -tile, three positive  $a$ -tiles, and two negative 1-tiles.



The opposite of  $a^2 - 3a + 2$  is  $-a^2 + 3a - 2$ .

You can also use inspection.  M E

$a^2$  is positive. The opposite of positive is negative.

The opposite of  $a^2$  is  $-a^2$ .

$3a$  is being subtracted. The opposite of subtracting  $3a$  is adding  $3a$ .

2 is being added. The opposite of adding 2 is subtracting 2.

The opposite of  $a^2 - 3a + 2$  is  $-a^2 + 3a - 2$ .

### Show You Know

What is the opposite of each expression? Justify your answer.

- $x$
- $5 - 3x$
- $7x^2 + 5x - 1$

**Strategies****Model It****Example 3: Subtract Polynomials**


Subtract  $2x + 3$  from  $3x - 4$ . Simplify your answer by combining like terms.

**Solution**


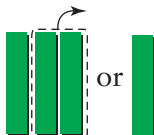
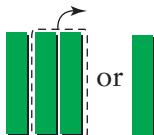
$$(3x - 4) - (2x + 3)$$

**Method 1: Use a Model**

You can use algebra tiles to model each polynomial.

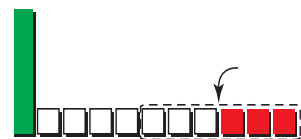
From  remove .

What other way could you model  $(3x - 4) - (2x + 3)$ ?

When you remove , you are left with  or .

You cannot yet remove  since there are no positive 1-tiles.

Add three zero pairs.



Now, you can remove .

You are left with  or  $x - 7$ .

$$(3x - 4) - (2x + 3) = x - 7$$

**Method 2: Add the Opposite**

One way to subtract a polynomial is to add the opposite terms.

$$\begin{aligned} &(3x - 4) - (2x + 3) \\ &= (3x - 4) + (-2x - 3) \\ &= 3x - 4 - 2x - 3 \\ &= 3x - 2x - 4 - 3 \\ &= 1x - 7 \text{ or } x - 7 \end{aligned}$$

How do you know the opposite of  $2x + 3$  is  $-2x - 3$ ?

Recall that when you subtract integers, you can add the opposite.

$$\begin{aligned} &(-2) - (+3) \\ &= (-2) + (-3) \\ &= -5 \end{aligned}$$
**WWW Web Link**

To learn more about adding and subtracting polynomials, go to [www.mathlinks9.ca](http://www.mathlinks9.ca) and follow the links.

**Show You Know**

**a)** Simplify the following expression. Model your solution.

$$(2x - 3) - (-x + 2)$$

**b)** Subtract and combine like terms.

$$(5x^2 - x + 4) - (2x^2 - 3x - 1)$$

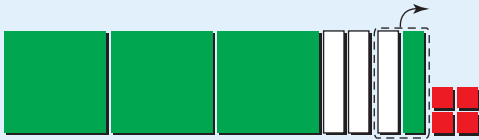
## Key Ideas

- You can add or subtract polynomials. You can use models to help simplify the expression.

$$(2x^2 - 3x) + (x^2 + x + 4)$$



Group like terms. Remove any zero pairs.



$$(2x^2 - 3x) + (x^2 + x + 4) = 3x^2 - 2x + 4$$

- The opposite of a polynomial is found by taking the opposite of each of its terms.

The opposite of  $-3x^2 + x + 1$  is  $3x^2 - x - 1$ .

- To subtract a polynomial, you can add the opposite terms.

$$(6x^2 - 3x + 4) - (x^2 - 3x + 2)$$

$$= (6x^2 - 3x + 4) + (-x^2 + 3x - 2)$$

$$= 6x^2 - x^2 - 3x + 3x + 4 - 2$$

$$= 5x^2 + 0x + 2$$

$$= 5x^2 + 2$$

$-3x + 3x = 0x$  or  $0$ , so it does not need to be included in the answer.

## Check Your Understanding

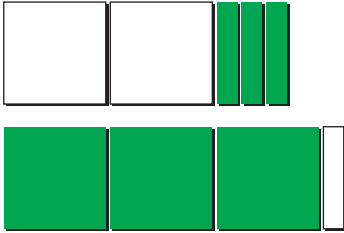
### Communicate the Ideas

- Jeanette and Tim find the answer to  $(3x^2 - 5x) - (4 - 2x)$ . Jeanette claims the simplified answer has three terms. Tim says it has only two terms. Who is correct? How do you know?
- What is the opposite of  $-x^2 + 2x - 3$ ? Use diagrams and then use symbols to determine the answer. Which method do you prefer? Why?
- Identify any errors in Mei's work and correct them.
 
$$\begin{aligned} &(-2x^2 + 7) - (3x^2 + x - 5) \\ &= (-2x^2 + 7) + (-3x^2 - x + 5) \\ &= -2x^2 - 3x^2 - x + 7 + 5 \\ &= 5x^2 - x + 12 \end{aligned}$$
- Create a situation in which the polynomials  $3x + 2$  and  $5x - 1$  are involved. In your situation, what does  $(3x + 2) + (5x - 1)$  represent?

## Practise

For help with #5 to #7, refer to Example 1 on page 191.

5. Which addition statement does the diagram model?






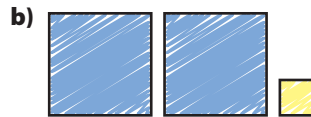
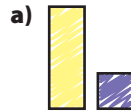
- A  $(2x^2 - 3x) + (3x^2 - x)$   
 B  $(-2x^2 + 3x) + (3x^2 + x)$   
 C  $(-2x^2 + 3x) + (3x^2 - x)$
6. Add the polynomials.
- a)  $(3x - 4) + (2x - 3)$   
 b)  $(-a^2 - 3a + 2) + (-4a^2 + 2a)$   
 c)  $(5p + 5) + (5p - 5)$   
 d)  $(2y^2 - 15) + (6y + 9)$
7. Perform the indicated operation and simplify by combining like terms.
- a)  $(-3x + 4) + (6x)$   
 b)  $(3n - 4) + (7 - 4n)$   
 c)  $(2b^2 - 3) + (-b^2 + 2)$   
 d)  $(5a^2 - 3a + 2) + (-4a^2 + 2a - 3)$

For help with #8 to #12, refer to Example 2 on pages 192–193.

8. What is the opposite of the expression represented by each diagram? Express your answer using both diagrams and symbols.



9. Let  represent  $x^2$ ,  represent  $x$ , and  represent 1. The same diagrams in yellow represent negative quantities. Determine the opposite of the expression represented by each diagram. Use both diagrams and symbols to express your answer.



10. What is the opposite of each expression?
- a)  $-9x$   
 b)  $5d + 6$   
 c)  $-2x^2 + 3x - 5$
11. What is the opposite of each expression?
- a)  $3x - 7$   
 b)  $4g^2 - 4g + 2.5$   
 c)  $v^2 + 8v - 1$

12. Which of the following represents the opposite of  $2x^2 - x$ ?

A  $-2x^2 - x$



D  $2x^2 + x$

For help with #13 to #15, refer to Example 3 on page 194.

13. Draw a diagram to model the subtraction statement  $(-3x^2 + 4x) - (-2x^2 - x)$ .

14. Simplify by combining like terms.

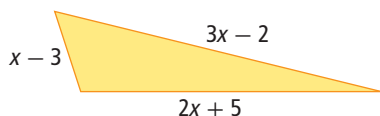
- a)  $(2x - 3) - (5x - 1)$
- b)  $(-3b^2 - 5b) - (2b^2 + 4b)$
- c)  $(5 - 6w) - (-2 - 3w)$
- d)  $(m + 7) - (m^2 + 7)$

15. Subtract.

- a)  $(8c - 3) - (-5c)$
- b)  $(-3r^2 - 5r - 2) - (r^2 - 2r + 4)$
- c)  $(y^2 - 5y) - (2y - y^2)$
- d)  $(6j^2 - 4j + 3) - (-2j^2 - 5)$

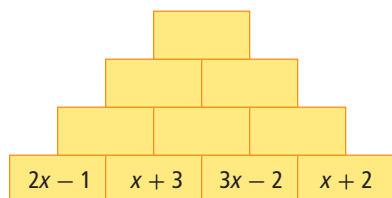
### Apply

16. A triangle has the dimensions shown.



- a) What does  $(x - 3) + (3x - 2) + (2x + 5)$  represent?
- b) Simplify the expression in part a).
- c) If  $x$  has a value of 5, what is the perimeter of the triangle? Did you use the expression in part a) or part b) to find this answer? Why?

17. Complete the addition pyramid. Find the value in any box by adding the expressions in the two boxes immediately below it.

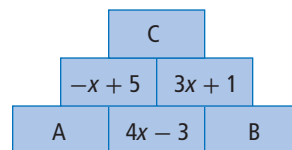


18. In Langley, British Columbia, you can rent a backhoe for \$399 per day and a bulldozer for \$550 per day. It costs \$160, round trip to move each piece of equipment back and forth to the job site.

- a) Write an expression for the total cost of renting the backhoe, before tax. Include transportation to and from the job site. What does your variable represent?
- b) What is an expression for the total cost of renting and moving the bulldozer? Use the same variable as in part a).
- c) What is an expression for the cost of renting both a backhoe and a bulldozer? Give your answer in its simplest form.
- d) What is an expression for the difference in cost between renting the backhoe and the bulldozer? Give your answer in simplest form.



19. Consider the addition pyramid shown below.



- a) Write an expression for box C. Do not simplify.
- b) Show how you can find the value for boxes A and B. Simplify your answers.



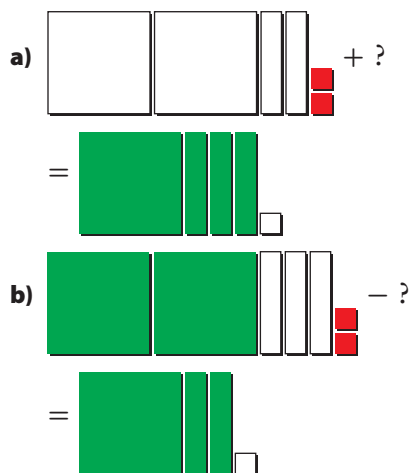
- 20.** The cost to print  $n$  copies of a book is  $15n + 2000$  dollars. The cost to ship  $n$  copies of the book is  $2n + 150$  dollars.
- What is an expression for the total cost to print and ship  $n$  copies of the book?
  - What is the actual cost to print and ship 600 copies of the book?
  - What does  $(15n + 2000) - (2n + 150)$  represent? Find a simpler expression for this subtraction statement.

- 21.** Describe any errors in Jorge's work and how you would correct each one.

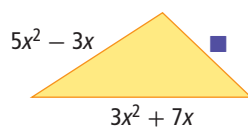
$$\begin{aligned} & (4p^2 - p + 3) - (p^2 + 3p - 2) \\ &= 4p^2 - p + 3 - p^2 - 3p - 2 \\ &= (4p^2 - p^2) + (-p - 3p) + (3 - 2) \\ &= 3p^2 - 3p + 1 \end{aligned}$$

- 22.** Simplify by combining like terms.
- $(6x - 7) + (3x - 1) + (x - 4)$
  - $(3a^2 - 4a) + (3a - 5) - (a^2 - 1)$
  - $(4t^2 - t + 6) - (t^2 + 2t - 4) + (2t^2 - 3t - 1)$
  - $(2x - 1.8) - (3.4x - 2.1) - (0.9x - 0.1)$

- 23.** Replace each question mark with algebra tiles to make a true statement.



- 24.** The perimeter of the triangle shown is  $12x^2 + 6x$ , in metres. Find a polynomial representing the missing side length.



- 25.** Your student council plans to thank 25 students and staff who have made special contributions to the

With Appreciation To  
**Mark and Lise**  
For All Your Help

school. Wooden plaques cost \$17.95. It costs \$0.12 per letter to engrave a message on the plaque. All costs are before tax.

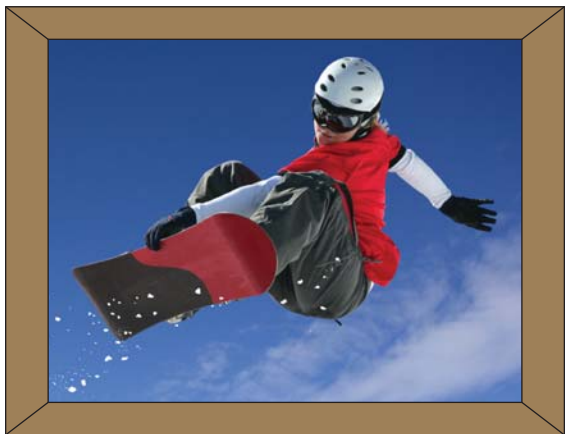
- Write an expression for the cost of engraving 25 plaques with the following message and the name of your school. *Thank you for your generous support.*
- Write an expression for the cost of buying and engraving the plaques.
- Write an expression for the cost of buying and engraving 25 plaques with an unknown number of letters.
- Show how to use the addition of polynomials to find the cost of 50 plaques if each plaque has the same number of letters and numbers.

### Extend

- 26.** Kiesha's dad is a Haisla artist. He makes his own prints and sells them on the Internet. He will ship the prints to purchasers anywhere in Canada. For large prints, he charges \$30 to ship one print plus \$7 for each additional print. For small prints, he charges \$20 for one print plus \$5 for each additional print.

- How much does her dad charge to ship two large prints?
- How much does he charge to ship four small prints?
- Write an expression to show how much he charges to ship an unknown number of large and small prints.

27. The length of the picture shown is 15 cm more than its width. The picture frame has a width of 4 cm. What is the minimum length of material needed to make the frame for this picture? Give your answer as a simplified expression.



28. A small manufacturer makes air quality monitoring kits for home use. The revenue, in dollars, from the sale of  $n$  kits can be shown by  $-n^2 + 3600n$ . The cost, in dollars, to make  $n$  kits is represented by  $-3n^2 + 8600$ . The manufacturer makes a profit if the cost subtracted from the revenue is positive.

- Write an expression to find the profit. Simplify your answer.
- Estimate and then calculate if the manufacturer will make a profit or suffer a loss after selling 20 test kits. Explain.

29. Simplify  $(2x + 4x + 6x + 8x + \dots + 2006x + 2008x) - (x + 3x + 5x + 7x + \dots + 2005x + 2007x)$ .

30. Mary is sewing two wall hangings. The length of one wall hanging is 56 cm greater than its width. The length of the other wall hanging is 15 cm greater than its width. Each of them has the same width. She is going to add a trim strip around each wall hanging. What is the total minimum length of trim she will need for both wall hangings?



"Camp, With Animals Nearby" by Annie Taipanak (1931-) of Baker Lake and Rankin Inlet, Nunavut.



"Hunting Caribou by Kayak" by Tobi Kreealak (1977-) of Baker Lake, Nunavut.

## Math Link

Try this number trick several times.

- How can you find the original number from the number in the last step?
- Use algebra to show why this number trick works.
- Find or make your own number trick. Use algebra to show why it works.

### Guess a Number

- Pick a number.
- Add 5.
- Double the sum.
- Subtract 10.