

5.2


Equivalent Expressions

Focus on...

After this lesson, you will be able to...

- use algebra tiles and diagrams to show whether expressions are equivalent
- identify equivalent expressions that are polynomials
- combine like terms in algebraic expressions

Materials

- concrete materials, such as algebra tiles 

Today's space program requires extensive use of algebra. Computer programs control shuttle flights and manipulate the Canadarm. They also control conditions inside the International Space Station. These programs use algebraic models, expressions, and equations. Where else in the real world is algebra used?

Explore Combining Like Terms

The astronauts on the space shuttle have a limited amount of living space. They eat, sleep, and relax in a rectangular space with a width of only about 3.2 m and a length that is 0.8 m greater than the width.

1. What is the length of the living space? How do you know?
2. Draw and label a diagram of the rectangular living space. Find the perimeter of the rectangle. How did you find the perimeter?
3. Draw another rectangle. The length is still 0.8 m greater than the width but you have no known value for the width. What could you use to represent the width of the rectangle? What would be an expression for the length of this rectangle?
4. Write an expression for the perimeter of the second rectangle.
5. How many terms are in your expression for the perimeter?
6. Use materials or a diagram to model your expression for the perimeter.
7. Rearrange your model so similar objects, shapes, or variables are all together. Combine the similar objects, shapes, or variables. What is an equivalent expression for the perimeter?

Reflect and Check

8. What do you think *like terms* means? Give examples to support your ideas.
9. How do you combine like terms in polynomials? Explain with examples.



Link the Ideas

Example 1: Identify Coefficients, Variables, and Exponents

For each expression, identify the coefficient, the variable(s), and the exponent of each variable.

- a) $3w$ b) a^2 c) $-4xy$ d) $-g$

Solution

Expression	Coefficient	Variable(s)	Exponents of the Variable(s)
a) $3w$	3	w	1
b) a^2	1	a	2
c) $-4xy$	-4	x and y	1 and 1
d) $-g$	-1	g	1

Literacy Link

Even if a term has two variables, it always has only one coefficient. Example: A term would be written as $-6xy$, not as $-2x3y$.

Show You Know

Give the coefficient, the variable(s), and the exponent of each variable.

- a) $3c^2$ b) $-x$ c) b d) $7st^2$

like terms

- terms that differ only by their numerical coefficients
- examples of like terms are
 - $3x$ and $-2x$
 - $6y^2$ and $-4y^2$
 - $-5xy$ and yx
 - 17 and -8

Example 2: Identify Like Terms

Identify the **like terms** in each group.

- a) $5b^2$ $3cb$ $-2b$ $7c$ $6b$
b) $3x^2$ $4xy$ $-2x^2$ $7x^2$ $\frac{1}{2}y$
c) $3pq$ 11 $-4q^2$ -3 pq

Solution

- a) $-2b$ and $6b$ are like terms. Both have a variable b with an exponent of 1. All the other terms are unlike.
- b) $3x^2$, $-2x^2$, and $7x^2$ are like terms. Each of them has a variable x with an exponent of 2. The other terms are unlike.
- c) $3pq$ and pq are like terms. Both have variables p and q , each with an exponent of 1. The terms 11 and -3 are also like terms.

Show You Know

- a) Give an example of three like terms.
- b) Identify the like terms in the following group: $6t$ $3s$ $6t^2$ $6st$ $-8s$

Example 3: Combine Like Terms

Combine like terms in each expression.

- a) $4x - 2x + 3 - 6$
- b) $2x^2 + 3x - 1 + x^2 - 4x - 2$
- c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$

Solution

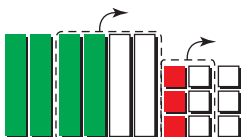
Method 1: Use a Model

- a) $4x - 2x + 3 - 6$

You can use algebra tiles to represent each term.



Group the tiles to form zero pairs and remove the pairs.



Write an expression for the remaining tiles.

$$2x - 3$$



So, $4x - 2x + 3 - 6 = 2x - 3$.

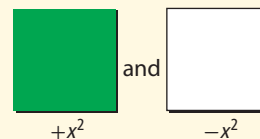
Like terms can be combined to simplify expressions.

Strategies

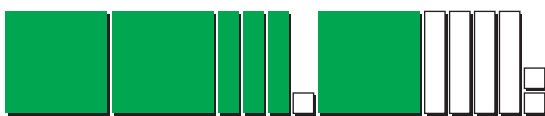
Model It

$+1$ and -1 have a combined value of zero and are called a zero pair.

The same is true for



- b) $2x^2 + 3x - 1 + x^2 - 4x - 2$



Group like terms and remove the zero pairs.



$$\begin{aligned} &2x^2 + 3x - 1 + x^2 - 4x - 2 \\ &= 3x^2 - 1x - 3 \\ &= 3x^2 - x - 3 \end{aligned}$$

Literacy Link

Any term with a variable having a coefficient of 1 can be written without its numerical coefficient. However, the sign must remain. Example:

$$\begin{aligned} -1x &= -x \\ +1x &= x \\ -1x^2 &= -x^2 \\ +1x^2 &= x^2 \end{aligned}$$

Coefficients without a sign are positive.

c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$



Group like terms and remove the zero pairs.



$$4 - x^2 + 2x - 5 + 3x^2 - 2x = 2x^2 - 1$$

Method 2: Use Symbols

a) Add or subtract the coefficients of like terms.

$$4x - 2x + 3 - 6 = 2x - 3$$

b) A polynomial with more than one term can be written in different orders. Rearrange by grouping like terms.

$$\begin{aligned} & 2x^2 + 3x - 1 + x^2 - 4x - 2 \\ &= 2x^2 + x^2 + 3x - 4x - 1 - 2 \\ &= 3x^2 - x - 3 \end{aligned}$$

c) $4 - x^2 + 2x - 5 + 3x^2 - 2x$
 $= -x^2 + 3x^2 + 2x - 2x + 4 - 5$
 $= 2x^2 - 1$

Literacy Link

In algebra, terms are often arranged in descending order by degree. For example, $-3y + 4y^2 - 1$ is written as $+4y^2 - 3y - 1$ or $4y^2 - 3y - 1$. This makes it easier to compare expressions. Answers are usually written this way.

Web Link

To learn more about the parts of a polynomial and combining like terms, go to www.mathlinks9.ca and follow the links.

Show You Know

Combine like terms.

a) $5x - 3x^2 + 2x - x^2$

b) $2x - 6 - 2x + 1$

c) $k - 2k^2 + 3 + 5k^2 - 3k - 4$

Key Ideas

- An algebraic expression is made up of terms. Each term can have any number of variables. Each variable has an exponent. A constant term, such as 9, has no variable.

- Like terms differ only by their numerical coefficients.

Like terms can be combined.

Like terms:

- $-7x$ and $3x$
- w^2 , $3w^2$, and $0.5w^2$
- 6 and 15

Unlike terms cannot be combined.

Unlike terms:

- $6x$ and $3x^2$
- m^2n and $4mn^2$
- 7 and $7d$

Term	Coefficient	Variable(s)	Variable's Exponent
$6p^2$	6	p	2
$-x^2y$	-1	x, y	2 for x , 1 for y

Check Your Understanding

Communicate the Ideas

- Using models, show how you know that $s - 5s$ combines to give $-4s$.
- Jean claimed that $3m^2 + 4m$ could be combined to give $7m^3$. Do you agree? Explain with diagrams.
- Most people would agree that $2 \text{ cats} + 5 \text{ cats} = 7 \text{ cats}$ and $7 \text{ cats} - 2 \text{ cats} = 5 \text{ cats}$. Use this information to support an argument for combining like terms and for being unable to combine unlike terms in algebra. Use examples with two different animals and two different variables.
- Does a number in front of a variable affect whether or not you have like terms? Explain using examples.

Practise

For help with #5 to #7, refer to Example 1 on page 184.

- What is the value of the coefficient and the number of variables for each term?
a) $-3z^2$ b) k c) 43
- Determine the value of the coefficient and the number of variables for each term.
a) $4d$ b) $-prt$ c) $-8fg^2$
- Use the following monomial expressions to answer the questions below:

$3x$	$4t$	x^2
$-ts$	xt	$2t^2$

- Which have a coefficient of 1?
- Which have two variables?
- Which have only one variable with an exponent of 1?
- Which have a coefficient of -1 ?

For help with #8 and #9, refer to Example 2 on page 184.

- Identify the like terms in each group.
a) $2a$ 5 $-7.1a$ $9b$ $-c$
b) $3m$ $-2ab$ $\frac{4}{3}m$ $-2ad$ m^2
c) -1.9 $6p^2$ 5 $-2p$ p^2

- Which terms are like terms in each group?

- $-2k$ 9 $104k$ $104f$ $-f^2$
- $\frac{1}{2}ab$ $0.5a$ $-4b$ ab^2 ab
- -5 $13d^2$ 5 $-10d$ d^2

For help with #10 to #12, refer to Example 3 on page 185.

- Collect like terms.
a) $3x - 2x^2 + x - 2x^2$
b) $-4 - 2n^2 - 3n + 3 + 2n^2$
c) $2q - 4q^2 - 2 + 3q^2 + 2 - 3q$
d) $-4c + 3 + 5c - 7$
e) $h^2 - 3h + 4h^2 + 2h$
f) $3j - 5 + 2j^2 - 1 + 2j - 3j^2$
- Simplify by collecting like terms.
a) $2d - 3d^2 + d^2 - 5d$
b) $y^2 + 2y - 2y^2 + y$
c) $-p + 4p^2 + 3 - 3p^2 - 5 + 2p$
d) $m - 4 + 6 + 3m$
e) $q^2 - 3q + 2q^2 - q$
f) $5w - 3 + w^2 - 2w - 4w^2 - 1$

12. Which expressions are equivalent to the simplified expression $-3x^2 + x - 4$?
- A $-4 + 3x^2 + x$
 B $x - 4 - 3x^2$
 C $x^2 + 2 - 4x^2 + 3x - 6 - 2x$
 D $-3 - 5x^2 + x + 1 + 2x^2$
 E $2x - 2 + x^2 - x - 4x^2 - 2$
 F $-4 - 3x - 3x^2 - 0 + 5x^2 + 4x - 6x^2$

Apply

13. Jessica and Taz are working on a measurement problem. Their calculations involve combining Jessica's measurement of 2 m and 32 cm with Taz's measurement of 1 m and 63 cm. Jessica claims you find the answer just like in algebra. Do you agree? Explain.

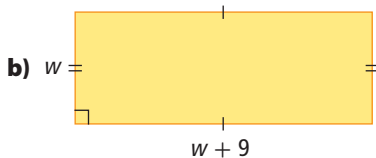
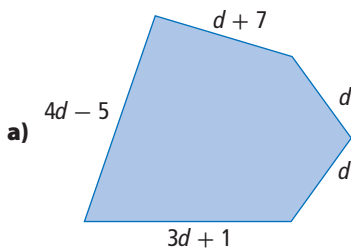
14. Describe a real-life situation that could be represented by each expression.

a) $m - 3$ b) $2p + 5$

15. For each of the following polynomials, write an equivalent expression with six terms.

a) $2p^2 - 3p + 2$ b) $-3x^2 + 5x - 4$
 c) $4r^2 - 2q^2 - 3qr$

16. Write an expression for the perimeter of each figure. Then, combine like terms if possible.



17. a) Draw a figure with a perimeter that is represented by $(5s) + (3s - 2) + (s + 6)$, where each value in parentheses represents the length of one side.
 b) Simplify the expression for the perimeter.

18. A student council decides to raise money by organizing a dance. The cost of a band is \$700. The student council decides to sell tickets at \$5 each.

- a) Write an expression for the profit that the student council would make. What does your variable represent?
 b) If 250 students pay to attend the dance, what is the profit?
 c) Estimate, then calculate, the minimum number of students who will need to buy tickets for the student council to make a profit.

19. A heating company charges \$60 per hour plus \$54 for a service call. Let n be the number of hours the technician works at your house.

- a) What expression represents the total cost of the job?
 b) What is the cost for a job requiring 2 h?
 c) The company charges half as much for a second technician. Write an expression showing the total cost if two technicians install a new furnace. Simplify your expression by combining like terms.

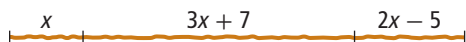
20. A publisher makes books for a number of distributors. For one book, the charge to the distributor is represented by a fixed cost of \$3000 plus \$16 per book.

- a) Write an expression for the cost that a distributor is charged for b copies of this book.
 b) How much do 600 books cost?
 c) What is the cost per book if 600 are ordered?
 d) What is the cost per book if 1000 are ordered?

21. Raj was told to write an expression equivalent to $3x - 8 - 5x + 9$.

$$\begin{aligned} & 3x - 8 - 5x + 9 \\ & = 3x - 5x - 8 + 9 \\ & = 2x - 1 \end{aligned}$$

- a) What errors did he make?
b) Show the correct response.
22. The diagram represents a piece of string.



- a) What is an expression for the total length of the string?
b) Combine like terms to get the simplest expression possible for the length of the string.

Extend

23. When would the expressions $x + y + 3$ and $x + w + 3$ be equal? How do you know?

24. A department store marks up wholesale prices 40% to get its retail or selling price.

- a) Complete the following table. The first row has been done for you.

Wholesale Price (\$)	Expression For Retail Price	Retail Price (\$)
8.00	$8 + (0.4)(8)$	11.20
12.00		
30.00		
x		

- b) How could you find the retail price if the wholesale price is $x + 10$ dollars? Show two ways to find the answer.
25. Zip Publishers will print posters for fundraising events for an initial cost of \$100 plus \$2 per poster. Henry's Printers charges \$150 plus \$1 per poster.
- a) Write an expression for each company, showing the total cost for any number of posters.
b) What is the cost of 125 posters from each company?
c) What is the total cost if you print 200 posters at each company? Show two different ways to find the answer.

Math Link

Refer to the Math Link for section 5.1 on page 182. Represent each item with a variable:

a = blender b = watch c = book
 d = soccer ball e = drum f = coffeemaker

- a) Rewrite all your combinations that add to \$100, using the letters a to f . Arrange each combination in alphabetical order. For example, $a + e + d + 3c$ would be written as $a + 3c + d + e$.
b) The example in part a) can be used to find other combinations. Notice that e has a value of \$40. What other items from the list have a value of \$40? By substituting into e the letter or letters that combine to a total value of \$40, you arrive at another answer. Do not forget to combine like terms and arrange each expression in alphabetical order. What other combinations can you find using substitution?
c) If you were asked to find combinations of the items that add to \$101, how could you use algebra to help you? Give two ways that algebra could help you.

