

The Language of Mathematics

Focus on...

After this lesson, you will be able to...

- use mathematical terminology to describe polynomials
- create a model for a given polynomial expression



The Great Wall is the world's largest human-made structure. It stretches over 6700 km across China. The wall was created by joining several regional walls.

algebra

- a branch of mathematics that uses symbols to represent unknown numbers or quantities

Similarly, mathematics is a developing science made up of several branches, including arithmetic, geometry, and **algebra**. It is a science that studies quantity, shape, and arrangement. As with any science, mathematics comes with its own unique language. The language of mathematics is universal. It can be understood anywhere in the world.

Look at the following paragraph. How much of it can you read? What languages do you think the paragraph contains?

古代希臘人相信, 如果將兩個物體同時拋下, 重的那個會先落地。加利略, 一個意大利人, 做實驗, 從比薩斜塔上擲球到地面。實驗顯示出不同的物體運行相同的距離所用的時間相同, 與它的質量/重量無關。它們之間的關係是公式 $y=ax^2$ 而不是像許多人預測的 $y=ax$ 。

Would the algebraic equations be any different if the paragraph was written in any other modern language?

Explore the Language of Algebra

- For the algebraic expression $5a + 4b$, what terminology can you use to describe the numbers 4 and 5, and the letters a and b ?
 - What terminology can you use to describe the expression $-7x^2$ and its parts?
- Make up a real-life situation and write an algebraic expression for it. What do the parts of your expression represent?
- Algebraic expressions can have different numbers of **terms**.

Number of Terms	Examples
1	5 $7x$ $-3ab$ $\frac{y}{2}$
2	$5 + x$ $3x^2 - 2$ $7xy + z^2$
3	$1 - x + y$ $2x^2 - 3x + 5$ $a + b + c$

Write other examples of expressions with one term, two terms, and three terms.

- A study of algebra includes working with **polynomials**. They are named by their number of terms. How do the names *monomial*, *binomial*, and *trinomial* relate to the number of terms in an expression?

What common words do you know that have prefixes of *mono*, *bi*, and *tri*?

term

- an expression formed from the product of numbers and/or variables
- $9x$ is a term representing the product of 9 (coefficient) and x (variable)
- a constant value, such as 5, is also a term

polynomial

- an algebraic expression made up of terms connected by the operations of addition or subtraction
- $3x^2 - 4$ has two terms. $3x^2$ and 4 are connected by the operation of subtraction.

Reflect and Check

- Look at the algebraic equations in the paragraph written in Chinese on the previous page. Use as much algebraic terminology as you can to describe them.

Language Link

The word *algebra* comes from Arabic. The word originated in Iraq. Around A.D. 830, Mohammad al-Khwarizmi of Baghdad wrote a book called *Hisab al-jabr w'al-muqabalah*. This book summarized Hindu understandings of equations and how to solve them. The whole title was too hard for some Europeans so they kept only the word *al-jabr*. We get the term *algebra* from that Arabic word.

WWW Web Link

For more information about the history of algebra, go to www.mathlinks9.ca and follow the links.

Literacy Link

Types of Polynomials

Some polynomials have specific names.

Monomial: one term

Binomial: two terms

Trinomial: three terms

All expressions with one or more terms are called *polynomials*.

Polynomial means many terms.

Link the Ideas

Example 1: Name Polynomials by the Number of Terms

For each expression, identify the number of terms and whether it is a monomial, binomial, trinomial, or polynomial.

- a) $4xy + 3$ b) $7a^2 - 2ab + b^2$
c) $5x^2 + y^2 + z^2 - x - 6$ d) 13

Solution

Expression	Number of Terms	Name
a) $4xy + 3$	2	binomial
b) $7a^2 - 2ab + b^2$	3	trinomial
c) $5x^2 + y^2 + z^2 - x - 6$	5	polynomial
d) 13	1	monomial

What are the two terms in $4xy + 3$?

Show You Know

For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.

- a) $5j^2$ b) $3 - m^2$
c) $ab^2 - ab + 1$ d) $-4x^2 + xy - y^2 + 10$

degree of a term

- the sum of the exponents on the variables in a single term (monomial)
- $3xz$, or $3x^1z^1$, has degree 2, since $1 + 1 = 2$
- $5x^2y$ and $-2b^3$ are terms of degree 3

degree of a polynomial

- the degree of the highest-degree term in a polynomial
- In $7a^2 - 3a$, the degree of the first term is 2, and the degree of the second term is 1. The highest degree is 2, so the degree of the polynomial is 2.

Example 2: Identify the Number of Terms and Degree of a Polynomial

What is the number of terms and the **degree** of each polynomial?

- a) $4x^2 + 3$ b) $7a^2 - 2ab + b^2$
c) $5x + z - 6$ d) 7

Solution

- a) The polynomial $4x^2 + 3$ has two terms. The term with the highest degree is $4x^2$. Its degree is 2. The degree of the polynomial is 2.

What degree does the constant term 3 have?

- b) The polynomial $7a^2 - 2ab + b^2$ has three terms. Each of the three terms has a degree of 2. The degree of the polynomial is 2.

When a variable has no exponent, the value of the exponent is 1. How do you know $2ab$ has a degree of 2?

Key Ideas

- Algebra is a branch of mathematics that uses symbols to represent unknown numbers or quantities. The symbols are often letters and are called variables.
- Polynomials are made up of terms. Terms are connected by addition or subtraction.
 $3x^2 + 2x - 7$ has three terms.
- Polynomials can have one or more terms. Some polynomials have specific names.

Name	Number of Terms	Example
monomial	1	$6x^2$
binomial	2	$3a^2 - 5$
trinomial	3	$-w^2 + 5w + 1$
polynomial	more than 3	$2s^2 - t^2 + st + 7t - 4$

- Each algebraic term has a degree. You can find the degree of a term by adding the exponents of the variable(s) in the term.
 $3x$ has degree 1. $-5x^2y$ has degree 3.
- A polynomial has the same degree as its highest-degree term.
 $x^2 + 5x - 7$ has degree 2.
 12 has degree 0.
- You can use models, such as algebra tiles and diagrams, to represent some polynomials.
 $2x^2 - 4x + 3$



Check Your Understanding

Communicate the Ideas

- Identify at least three mathematical words or phrases you could use to refer to the polynomial $-5 + x^2$.
- Sonja and Myron are discussing the algebra tile model shown below.



Sonja says, “The model represents the expression $3x^2 + x + 2$.” Myron claims, “It represents $3x^2 - x - 2$.” Who is correct? How do you know?

3. Give two examples of a polynomial that satisfies all statements.
- consists of two terms
 - contains two variables
 - has degree 2
 - one term is of degree 1 and has a coefficient of 1
4. When is it acceptable not to write the 1 in an algebraic expression?
When must you write the 1? Give examples.

Practise

For help with #5 to #7, refer to Example 1 on page 176.

5. For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.
- a) $3x^2 - 5x - 7$ b) $-11a$
 c) $c^2 + cf + df - f^2$ d) 8
6. What is the number of terms and what is a name for each expression?
- a) n b) $6 + 4x - x^2$
 c) 0 d) $p^2 + 3pq$
7. Refer to the polynomials below to answer each question.

$6x$	-15
$3x - y$	$4c^2 - cd$
$7 + a + b$	$3m^2 - 4mn - 9n^2 + 1$

- a) Which ones are monomials?
 b) Which ones are trinomials?
 c) Which ones have two terms?

For help with #8 to #10, refer to Example 2 on pages 176–177.

8. For each polynomial, what is the degree and number of terms?
- a) $4 - b$ b) $fg + 2g$
 c) $8x^2 - xy - y^2$
9. State the degree and number of terms for each polynomial.
- a) $3xy + 1$ b) $11k^2 + 7k - 5$
 c) 6

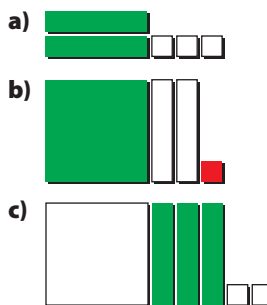
10. Refer to the polynomials below to answer each question.

$3b^2$	$2 + p$
$4st + t - 1$	$2x^2 - y^2$

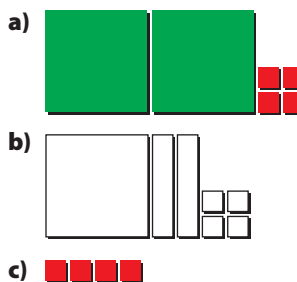
- a) Which ones are binomials?
 b) Which ones have degree 2?
 c) What is the variable in the monomial?
 d) Which polynomials have a constant term?

For help with #11 to #14, refer to Example 3 on page 177.

11. What expression is represented by each set of algebra tiles?



12. Write the expression represented by each set of algebra tiles.



13. Model each polynomial.

- a) $x^2 + x - 1$
- b) $3x + 2$
- c) $-2x$

14. Use a model to represent each polynomial.

- a) $-x^2 + 3$
- b) $2x^2 - 3x$
- c) 8

Apply

15. Represent each of the following with a diagram and an expression.

- a) binomial
- b) monomial of degree 1
- c) monomial of degree 2 with a coefficient of 9
- d) polynomial with four terms that is of degree 2

16. Use your knowledge of algebra tiles to answer the following questions.

- a) How are the dimensions of a 1-tile and an x -tile related?
- b) The rectangle shown was formed using an x^2 -tile and three x -tiles. What is an expression for the length of the rectangle?



17. Write an algebraic expression for each of the following.




- a) the product of 6 and x
- b) the sum of $2x$ and 3
- c) the length of the rectangle below, which is made from algebra tiles



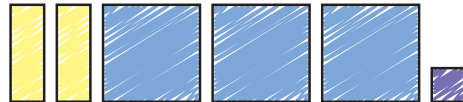
18. Make a model of an algebraic expression that includes at least one x^2 -tile, at least two x -tiles, and two 1-tiles. Use materials or a diagram. Then, use symbols to show your expression. What type of polynomial is it?

19. For the polynomial $6x^2 - 5$, state the following.

- a) number of terms
- b) coefficient of the first term
- c) number of variables
- d) degree of polynomial
- e) constant term

20. Let  represent x^2 ,  represent x , and  represent 1. The same diagrams in yellow represent negative quantities.

a) What is an expression for the polynomial shown?



b) Make up a trinomial. Draw diagrams to represent your trinomial.

21. Write each statement as an algebraic expression. Include what your variables represent.

- a) Eight and a number are added together.
- b) Omar has some money in his wallet. How much money does he have after a friend gives him \$5?
- c) A page is 4 cm longer than its width.
- d) The product of a number and 5 is increased by 2.
- e) The result of 3 times the number of people decreased by 21.

22. Describe a situation that could be modelled by each given polynomial.

- a) $3x + 5$
- b) $10 - x$

- 23.** Marion gives French lessons in the evening. She charges \$20 for adults and \$15 for children. The expression $20a + 15c$ represents her earnings.
- What do the variables a and c represent?
 - How much does Marion make if she gives lessons to four adults and nine children? Show your work.
 - Write a new expression for Marion's earnings if she charges \$3 more for adults and \$2 more for children.
- 24.** Tickets for a school concert are \$10 for adults and \$5 for students. Write an expression that shows the total income for the school concert. Tell what your variables represent.
- 25.** A hockey league awards teams two points for a win, one point for a shoot-out loss, and no points for a loss in regulation time.

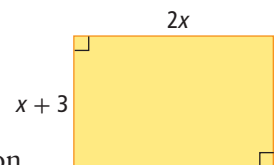


- Write an algebraic expression to represent the total points for a hockey team.
- What variable(s) did you use? Indicate what each variable represents.
- In the first 20 games of the season, Team A had 12 wins and 4 shoot-out losses. How many losses in regulation time did the team have?
- What were the total points for Team A?
- Team A was tied with Team B after 20 games. However, Team B had a different record than Team A. Show two possible records for Team B. Use your expression to show that the two hockey teams had the same number of total points.

- 26.** A banquet hall can be rented for parties. An expression for the rental cost is $5n + 75$, where n is the number of people.
- What type of polynomial is $5n + 75$, and what is its degree?
 - What could the numbers 5 and 75 represent?
 - How much does it cost to rent the banquet hall for 150 people?

Extend

- 27.** On a true/false test, there is a penalty for incorrect answers. Miranda's teacher advises the students not to guess at any of the 25 questions. The teacher awards 2 points for a correct answer, -1 point for a wrong answer, and 0 points if the question is not answered.
- Write a polynomial to represent a student's score on this test.
 - What are the maximum and minimum scores possible on this test? Explain.
 - What are all of the possible scores if Miranda got 20 questions correct? Explain.
- 28.** What is the degree of $xy - abx + cdy - qr - prqz$ if x , y , and z are variables and a , b , c , d , p , q , and r are coefficients?
- 29.** Ricardo draws the following rectangle with dimensions in metres.



- What is an expression for the perimeter of the rectangle?
- Write an equation showing how the length and width of the diagram would be related if the dimensions given were for a square.
- Solve your equation in part b) to find the value of x . Show your work.

- 30.** Create a polynomial satisfying the following conditions:
- contains three variables
 - has three terms
 - is of degree 2
 - has a constant term, 3

31. Deidra is training for a triathlon. From her training, she knows that she can swim at 1.3 km/h, cycle at 28 km/h, and run at 12 km/h.

- a)** Write the formula distance (d) = speed \times time using variables of your choice for speed and time. Tell what each variable represents.

The distance for each leg of the competition is different. In the table, this difference is shown using subscripts.

- b)** Use your formula to complete the table.

Part of Race	Distance (km)	Speed (km/h)	Time (h)
swim	d_1	1.3	$\frac{d_1}{1.3}$
cycle	d_2	28.0	
run	d_3	12.0	

- c)** Write a trinomial to model Deidra's total time for a triathlon.
- d)** A triathlon includes a 1.5-km swim, a 40-km cycle, and a 10-km run. How long will it take Deidra to complete this triathlon? What assumptions are you making?
- e)** If Deidra could maintain the same speeds, how long would it take her to complete a triathlon that is a 3.8-km swim, 180-km cycle, and 42.2-km run?

WWW Web Link

The Ironman Canada Triathlon in Penticton, B.C., involves a 3.8-km swim, 180-km cycle, and 42.2-km run. The times of recent winners are impressive. Top men's times are around 8 h, while women's times are around 9 h. For information about the Ironman Canada Triathlon and about the history of the Ironman competition, go to www.mathlinks9.ca and follow the links.



Math Link

You want to be a contestant on a game show. In order to get on the show, you must show how to spend exactly \$100 by choosing from the items shown.

You may purchase some or all of the six items, and as many of a single item as necessary.

- a)** Find at least six answers that would get you on the game show.
- b)** Write an algebraic expression for one of your combinations in part a). What is an equation for this same combination?
- c)** Is it possible to spend \$100 choosing all different items? Explain.

