1.3 Surface Area

Focus on...

After this lesson, you will be able to…

- **determine the area of overlap in composite 3-D objects**
- **find the surface area for composite 3-D objects**
- **solve problems involving surface area**

Red blood cells are the shape of very tiny disks. They have a thickness of 2.2 microns and a diameter of 7.1 microns. A micron is another term for a micrometre—one millionth of a metre. Red blood cells absorb oxygen from the lungs and carry it to other parts of the body. The cell absorbs oxygen through its surface.

The disease multiple mycloma causes the red blood cells to stick together. How would this affect the **surface area** of the cells?

surface area

- the sum of the areas of all the faces of an object

Materials

- small disks or pennies
- small boxes or dominoes

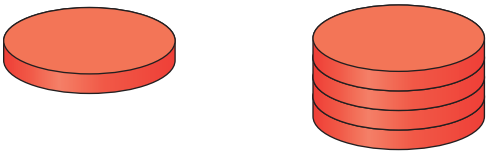
Explore Symmetry and Surface Area

1. a) Use a small disk to represent a single red blood cell. Estimate the surface area of the disk.

b) Stack four disks. Estimate the surface area of the stack of disks.

c) How did you estimate the surface area for parts a) and b)? Compare your method of estimation with your classmates' methods.

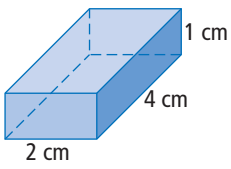
d) How does the total surface area of the four separate disks compare to the surface area of the four stacked disks? By what percent did the total surface area decrease when the disks were stacked?



Literacy Link

An object that is made from two or more separate objects is called a *composite object*.

2. Some medicine is shipped in small boxes that measure 1 cm by 4 cm by 2 cm. Six boxes are wrapped and shipped together. Working with a partner, use models to help answer the following questions.



a) If the arrangement of the six boxes must form a rectangular prism, how many arrangements are possible?

b) The cost to ship a package depends partly on total surface area. Would it be cheaper to ship the boxes in part a) individually, or wrapped together in plastic? If you wrapped the boxes together, which arrangement do you think will cost the least to ship? Explain.

3. You want to waterproof a tent. You need to determine the surface area of the tent's sides and ends to purchase the right amount of waterproofing spray. You do not have to waterproof the bottom. Calculate the surface area. Give your answer to the nearest tenth of a square metre.

How can you use the Pythagorean relationship to find the dimension for the tent's sides?

Reflect and Check

4. How can symmetry help you find the surface area in each of the three situations? Explain.

5. How does the surface area of a composite object compare with the sum of the surface areas of its separate parts? Explain.

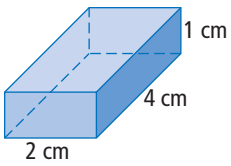
Link the Ideas

Different formulas can be used to find the surface area of a rectangular prism or a cylinder. There is one formula that works for both:

Surface Area = 2(area of base) + (perimeter of base) × (height)

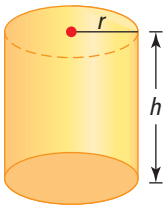
SA_{prism} = 2(area of base) + (perimeter of base) × (height)
= 2(4 × 2) + (4 + 2 + 4 + 2) × 1
= 16 + 12
= 28

The surface of this prism is 28 cm^2.



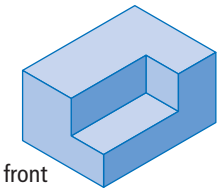
Using this same approach, the formula for the surface area of a cylinder is

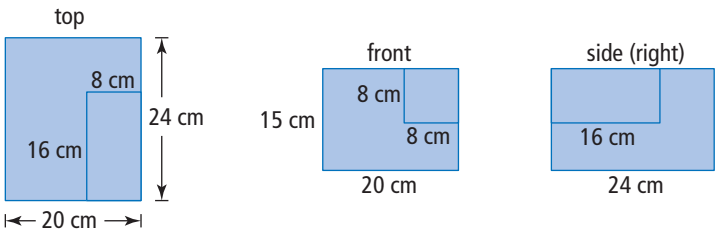
$SA_{cylinder}$ = 2(area of base) + (perimeter of base) × (height)
= $2(\pi r^2) + (2\pi r)h$
= $2\pi r^2 + 2\pi rh$



Example 1: Calculating Surface Area of a Solid

Consider the solid shown, in which all angles are right angles.

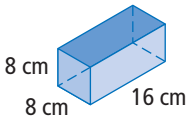




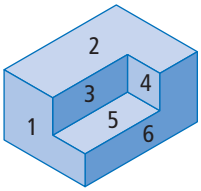
a) What are the dimensions of the cutout piece?
b) What is the total surface area of the solid?

Solution

a) The cutout notch is a right rectangular prism. The dimensions of the notch are 8 cm by 8 cm by 16 cm.



b) *Method 1: Find the Surface Area of Each Face*
You need to find the area of nine faces, including the faces of the notch. Number the faces to help keep track of the faces you have completed. Let the left face be #7, the back #8, and the bottom #9.



Why do you subtract 8 × 8 in the calculation for face 1?

Face	Calculation	Surface Area (cm²)
1	15 × 20 − (8 × 8)	236
2	20 × 24 − (8 × 16)	352
3	8 × 16	128
4	8 × 8	64
5	8 × 16	128
6	15 × 24 − (8 × 16)	232
7 (left side)	15 × 24	360
8 (back)	15 × 20	300
9 (bottom)	20 × 24	480
	Total Surface Area:	2280

The total surface area of the solid is 2280 cm².

Method 2: Use Symmetry

Calculate the surface area of only certain faces.

face 9 (bottom): $20 \times 24 = 480$
face 8 (back): $15 \times 20 = 300$
face 7 (left side): $15 \times 24 = 360$
Total of 3 faces: 1140

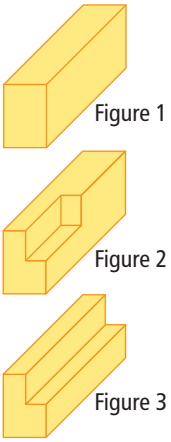
Notice that, by symmetry, opposite faces match.
face 2 + face 5 = face 9
face 1 + face 4 = face 8
face 6 + face 3 = face 7
You can obtain the surface area by doubling the area for face 9 + face 8 + face 7.
$1140 \times 2 = 2280$

The surface area of the solid is 2280 cm^2.

Why could the following be used to calculate the surface area?
$SA = 2(15 \times 20) + 2(15 \times 24) + 2(20 \times 24)$

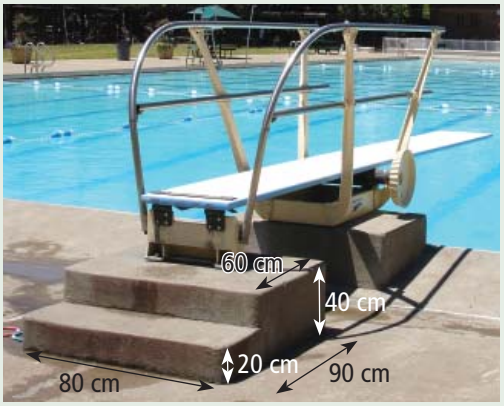
Did You Know?

If you cut a right rectangular piece out of one corner of a rectangular prism (Figure 2), the surface area does not change from that of the original prism (Figure 1). The surface area does change if the cutout extends across the solid (Figure 3). Explain why.



Show You Know

A set of concrete steps has the dimensions shown. Estimate and then calculate the surface area of the faces that are not against the ground. What is the area of the surface that is against the ground? Explain your answer.

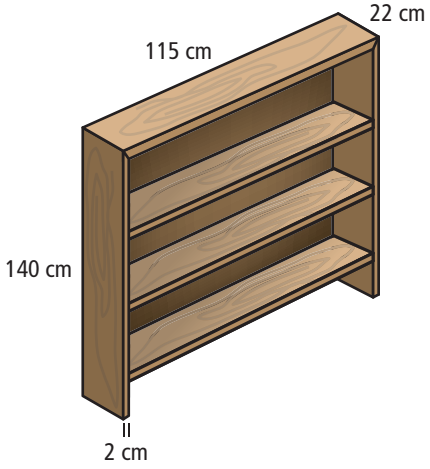


Example 2: Painting a Bookcase

Strategies
Make an Assumption

Raubyn has made a bookcase using wood that is 2 cm thick for the frame and shelves. The back is thin plywood. He wants to paint the entire visible surface. He will not paint the back, which stands against a wall.

a) What assumptions could you make about how the bookcase is painted?
b) What surface area does Raubyn need to paint?



Solution

a) Assumptions could include:
He paints the undersides of the three shelves.
The shelves are set inside the ends of the bookcase.
He paints the visible or inside back surface.
He does not paint the area of the base on which the bookcase stands.
Raubyn paints the bookcase after it is assembled.

Why would this assumption make a difference?

b) Group similar surfaces together.
Group 1: underside of top, and top and bottom of each of the three shelves.
Surface area $= 7 \times 111 \times 22$
$= 17\ 094$

Why is this measurement 111 cm, rather than 115 cm?

Group 2: outside of top and sides.
Surface area $= 22 \times 115 + 2(22 \times 140)$
$= 8690$

Group 3: back of bookcase that shows inside and front edges of the three shelves.
Surface area $= 111 \times 138$
$= 15\ 318$

Group 4: front edges of top and sides.
Surface area $= 2(2 \times 138) + 2 \times 115$
$= 782$

This measurement is 138, rather than 140, because the top piece is 2 cm thick. Notice that no surface area was subtracted to account for the back edges of the shelves, and none was added to account for the front edges. Using symmetry, explain why this works.

Total surface area:
$17\ 084 + 8690 + 15\ 318 + 782 = 41\ 884$
The surface area Raubyn needs to paint is $41\ 884\ cm^2$.

WWW Web Link
For information on how to calculate the surface area of different shapes, go to www.mathlinks9.ca and follow the links.

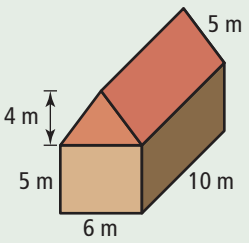
Show You Know

Consider the building shown.

a) Estimate the outside surface area of the building.

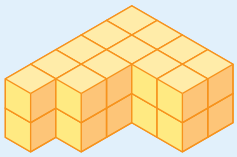
b) Calculate the outside surface area. Determine your answer two different ways.

c) Which method do you prefer? Why?



Key Ideas

- To determine the surface area of a composite 3-D object, decide which faces of the object you must consider and what their dimensions are.
- There are several ways to determine the surface area of an object.
  - Determine the area of each face. Add these areas together.
  - Use symmetry to group similar faces. Calculate the area of one of the symmetrical faces. Then, multiply by the number of like faces. This reduces the number of faces for which you need to calculate the surface area.



The top of this object has an area of 13 square units. The bottom must have the same area.

  - Consider how the shape is made from its component parts. Determine the surface area of each part. Then, remove the area of overlapping surfaces.

Check Your Understanding

Communicate the Ideas

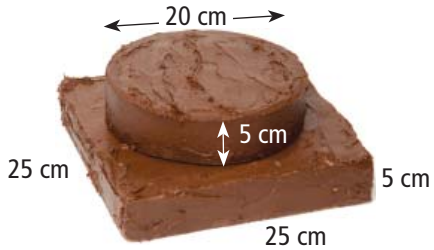
1. Build two different solid objects each using 24 interlocking cubes.

a) Explain how symmetry could help you determine the surface area of one of your objects.

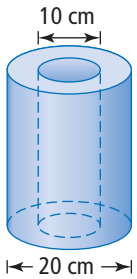
b) Slide the two objects together. What is the area of overlap between the objects?

c) How does the overlap affect the total surface area of your composite object?

2. Nick makes a two-layer cake. Instead of icing, he puts strawberry jam between the two layers. He plans to cover the outside of the cake with chocolate icing. Describe how he can calculate the area that needs icing.



3. Explain how you would calculate the surface area of the object shown.

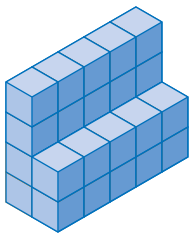


Practise

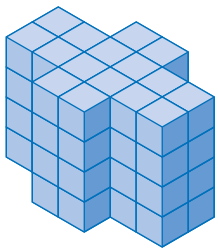
For help with #4 to #7, refer to Example 1 on pages 28–29.

4. Each object has been constructed from centimetre cubes. Estimate and then calculate the surface area.

a)

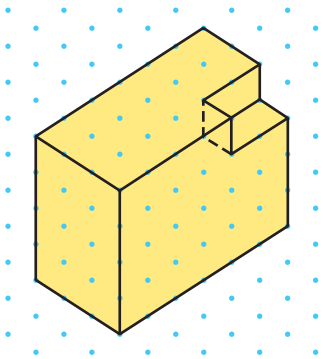


b)

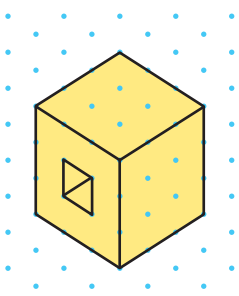


5. The following objects have been drawn on isometric dot paper where the distance between dots is 2 cm. Determine the surface area of each object.

a)

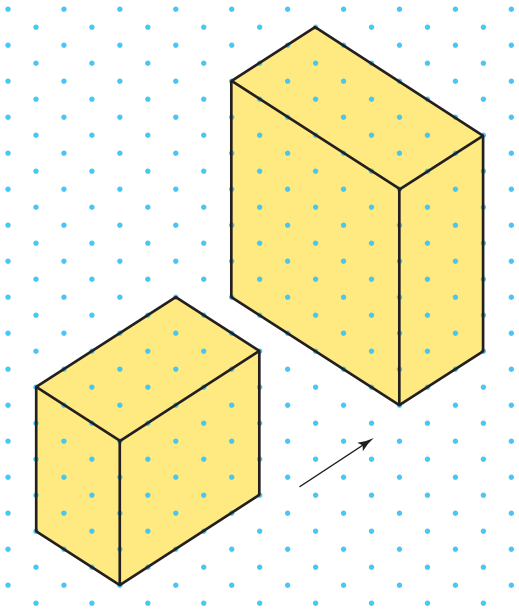


b)



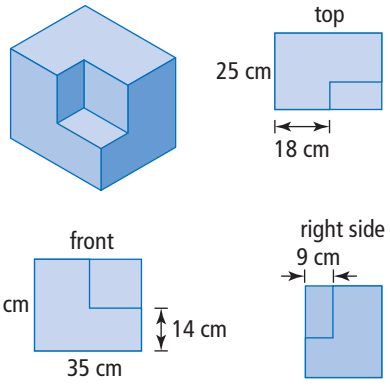
Note: The hole extends all the way through the block.

6. a) If you build the rectangular solids and slide them together as shown, what is the area of the overlap? Assume the dots are 1 cm apart.



b) What is the surface area when the solids are together?

7. Examine the solid and its views. All angles are right angles.

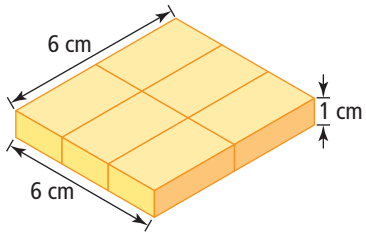


a) What are the dimensions of the cutout piece?

b) Explain how cutting out the corner piece will affect the surface area of the original rectangular solid.

For help with #8 and #9, refer to Example 2 on pages 29–30.

8. Six small boxes, all the same size, have been arranged as shown.



a) What are the dimensions of a single box?

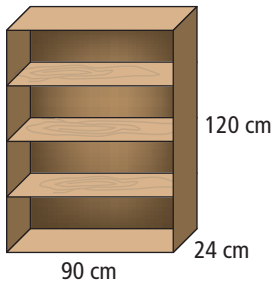
b) What is the surface area for the arrangement of the six boxes?

c) What is the ratio of the answer in part b) to the total surface area of the six separate boxes?

WWW Web Link

To see how surface area changes when a composite object is broken apart, go to www.mathlinks9.ca and follow the links.

9. Examine the bookshelf. It is constructed of thin hardwood. The top, bottom, and all three shelves are the same size. There is an equal distance between the top, the shelves, and the base.



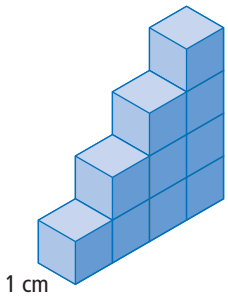
a) What is the surface area of one shelf? Include both sides, but ignore the edges.

b) What is the total surface area of the bookcase?

c) What is the fewest number of surfaces for which you need to find the surface area in order to answer part b)?

Apply

10. Use centimetre cubes to build the object shown.

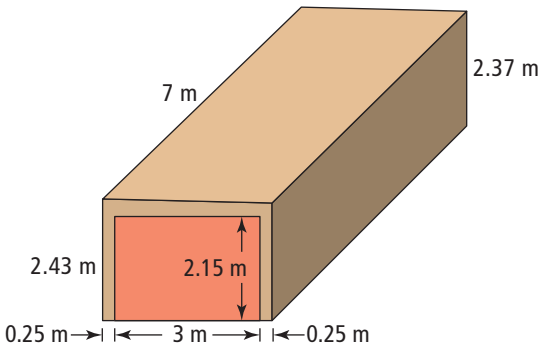


a) What is the object's surface area?

b) Take the same ten cubes and build a rectangular prism. Estimate and then calculate whether the surface area remains the same. Explain with examples.

11. List places or situations in which surface area is important. Compare your list with those of your classmates.

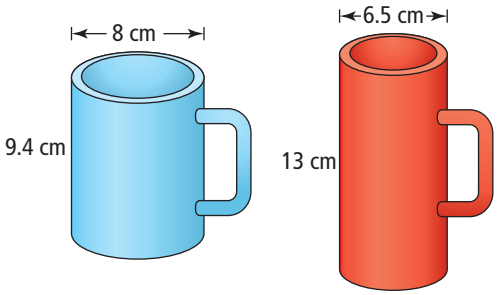
12. Consider this drawing of a garage. The left side of the garage is attached to the house.



a) What is the difference in height between the left-hand and right-hand sides of the garage? Explain why you would want a slight slant to a roof?

b) Given that the house is attached to the left side of the garage, what is the surface area of the garage to the nearest hundredth of a square metre? What assumption(s) did you make in answering this question?

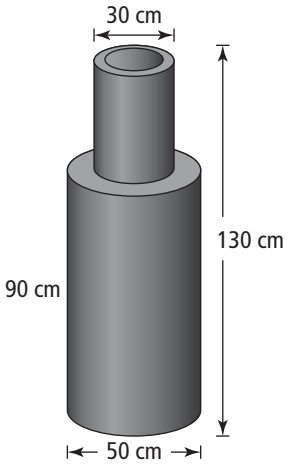
13. A mug for hot beverages is to be designed to keep its contents warm as long as possible. The rate at which the beverage cools depends on the surface area of the container. The larger the surface area of the mug, the quicker the liquid inside it will cool.



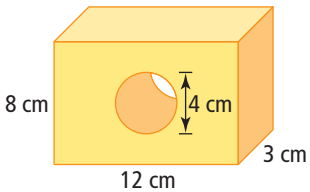
a) What is the surface area of each mug? Assume that neither has a lid.

b) Which is the better mug for keeping drinks warm? Justify your answer.

14. A chimney has the dimensions shown. What is the outside surface area of the chimney? Give your answer to the nearest hundredth of a square metre.



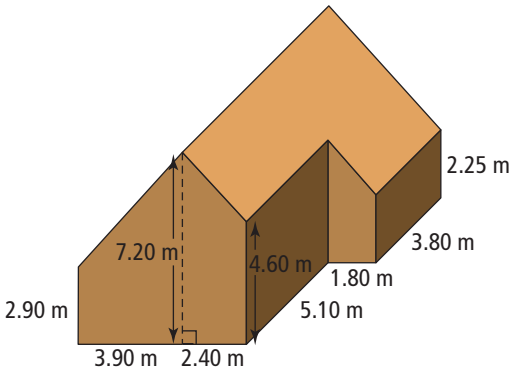
15. Twila made the object shown.



a) How can you use symmetry to help find the surface area of this object?

b) What is the surface area?

16. You are planning to put new shingles on the roof of the home shown.

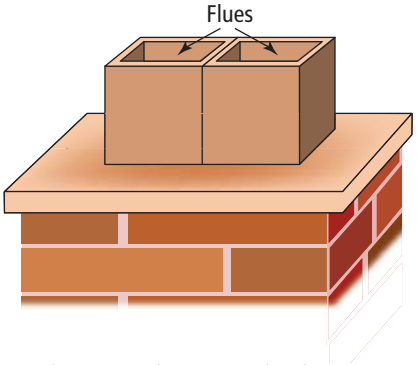


a) How many times would you need to use the Pythagorean relationship in order to find the area of the roof of the building shown in the diagram?

b) What is the area of the roof that you cannot see in this figure, assuming that it is a rectangular roof?

c) One bundle of shingles covers approximately 2.88 m^2 and costs $26.95. What does it cost for shingles to cover the roof?

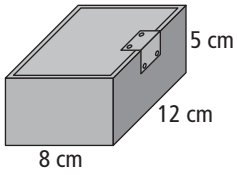
17. The hollow passages through which smoke and fumes escape in a chimney are called flues. Each flue shown is 2 cm thick, 20 cm high, and has a square opening that is 20 cm by 20 cm.



a) What are the outside dimensions of the two flues?

b) If the height of each flue is 30 cm, what is the outside surface area of the two flues? Hint: Do not forget the flat edges on top.

18. A small metal box is shown. What is the inside surface area of the box? What assumptions did you make in finding your answer?



19. A party planner buys two plain cakes for a meal she is planning. One cake is square and the other is round. Both cakes are 6 cm thick. The square cake measures 25 cm along each edge. The round cake has a diameter of 25 cm.

a) Sketch and label a diagram of each cake.

b) Show how to make four cuts to create eight equal pieces for each cake.

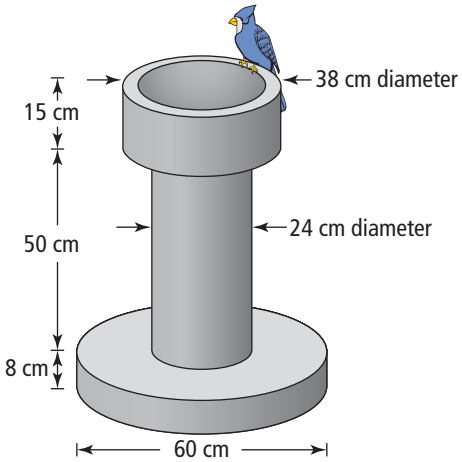
c) Estimate and then calculate how much the surface area increases after each cake is cut and the pieces are slightly separated.

Extend

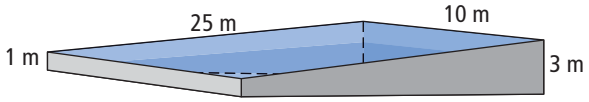
20. Explain how surface area of individual grains of rice may affect the boiling of a cup of uncooked rice. Assume you have two kinds of rice. One has small grains and the other has larger ones. Consider each grain of rice to be cylindrical.

21. An elephant's ears are one of nature's best examples of the importance of surface area in heating and cooling. Research this phenomenon or another one that interests you. Write a brief report outlining the importance of surface area in heating and cooling. (Two other possible topics are why radiators have complex internal shapes and how a cactus minimizes surface area.)

22. The plan for a concrete birdbath is shown below. The bowl is a cylinder with a depth of 10 cm. If the bowl has a diameter of 30 cm, what is the exposed surface area of the birdbath, including the pillar and pedestal?



23. A swimming pool measures 25 m long and 10 m wide. It has a shallow end that is 1 m deep and gradually slopes down to a depth of 3 m at the deep end. The inside walls of the pool need repainting. Calculate the total area of the surfaces to be painted, to the nearest square metre.



Math Link

Your design company wants to create a new product that will have a design printed on it. Your project team has suggested playing cards, business cards, memo pads, and sticky notes. Choose one of these items.

a) What are the dimensions of your pack of cards or pad of paper?

b) What is the surface area of your pack of cards or pad of paper?