1.2

Rotation Symmetry and Transformations

Focus on...

After this lesson, you will be able to...

- tell if 2-D shapes and designs have rotation symmetry
- give the order of rotation and angle of rotation for various shapes
- create designs with rotation symmetry
- identify the transformations in shapes and designs involving line or rotation symmetry

Some 2-D shapes and designs do not demonstrate line symmetry, but are still identified as having symmetry. The logo shown has this type of symmetry. What type of transformation can be demonstrated in this symbol?

Explore Symmetry of a Rotation

Materials

- scissors
- tracing paper

Look carefully at the logo shown.

1. The logo has symmetry of rotation. What do you think that means?

2. Copy the logo using tracing paper. Place your drawing on top of the original figure. Put the point of your pencil on the tracing paper and rotate the design until the traced design fits perfectly over the original design.

a) Where did you have to put your pencil so that you were able to rotate your copy so that it fit over the original? How did you decide where to put your pencil? Explain why it is appropriate that this point is called the **centre of rotation**.

b) How many times will your tracing fit over the original design, in one complete turn?

c) Approximately how many degrees did you turn your tracing each time before it overlapped the original?

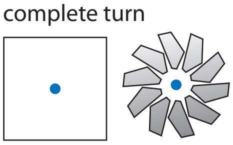
3. Work with a partner to try #2 with some other logos or designs.

centre of rotation

- the point about which the rotation of an object or design turns

rotation symmetry

- occurs when a shape or design can be turned about its centre of rotation so that it fits onto its outline more than once in a complete turn



Reflect and Check

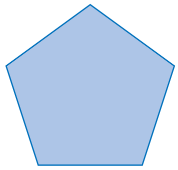
4. What information can you use to describe **rotation symmetry**?

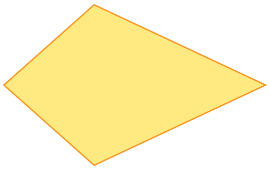
Link the Ideas

Example 1: Find Order and Angle of Rotation

For each shape, what are the order of rotation and the angle of rotation? Express the angle of rotation in degrees and as a fraction of a revolution.

a) 

b) 

c) 

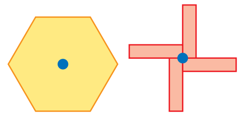
Solution

Copy each shape or design onto a separate piece of tracing paper. Place your copy over the original, and rotate it to determine the order and angle of rotation.

	Order of Rotation	Angle of Rotation (Degrees)	Angle of Rotation (Fraction of Turn)
a)	2	$\frac{360°}{2} = 180°$	$\frac{1 \text{ turn}}{2} = \frac{1}{2}$ turn
b)	5	$\frac{360°}{5} = 72°$	$\frac{1 \text{ turn}}{5} = \frac{1}{5}$ turn
c)	1	360°	1 turn

The figure in part c) does not have rotational symmetry.

order of rotation
- the number of times a shape or design fits onto itself in one complete turn



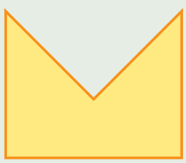
6 4

angle of rotation
- the minimum measure of the angle needed to turn a shape or design onto itself
- may be measured in degrees or fractions of a turn
- is equal to 360° divided by the order of rotation

Show You Know

For each shape, give the order of rotation, and the angle of rotation in degrees and as a fraction. Which of the designs have rotation symmetry?

a) 

b) 

c) 

Did You Know?

The Métis flag shown in part a) is a white infinity symbol on a blue background. The infinity symbol can represent that the Métis nation will go on forever. It can also be interpreted as two conjoined circles, representing the joining of two cultures: European and First Nations.

Example 2: Relating Symmetry to Transformations

Visualize the translation and rotation of the figures. How does this help you determine the type of symmetry that they demonstrate?

Examine the figures.

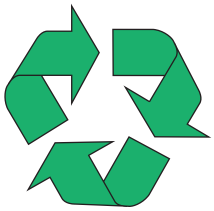


Figure 1

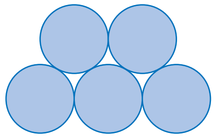


Figure 2

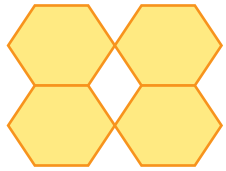


Figure 3

a) What type of symmetry does each figure demonstrate?
b) For each example of line symmetry, indicate how many lines of symmetry there are. Describe whether the lines of symmetry are vertical, horizontal, or oblique.
c) For each example of rotation symmetry, give the order of rotation, and the angle of rotation in degrees.
d) How could each design be created from a single shape using translation, reflection, and/or rotation?

Solution

The answers to parts a), b), and c) have been organized in a table.

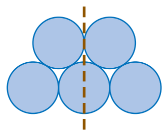
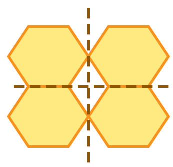
	Figure 1	**Figure 2**	**Figure 3**
a) Type of symmetry	rotation	line	rotation and line
b) Number and direction of lines of symmetry	No lines of symmetry 	Total = 1: vertical 	Total = 2: 1 vertical 1 horizontal 
c) Order of rotation	3	1	2
Angle of rotation	$\frac{360°}{3} = 120°$	360°	$\frac{360°}{2} = 180°$

Figure 2 does not have rotational symmetry

d) Figure 1 can be created from a single arrow by rotating it $\frac{1}{3}$ of a turn about the centre of rotation, as shown.

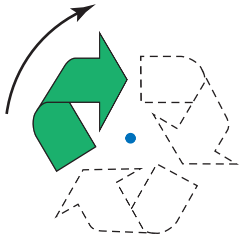
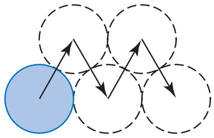
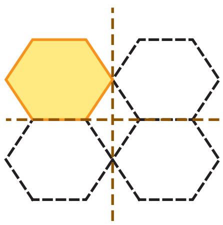


Figure 2 can be created from a single circle by translating it four times.



How could you use reflection to create this figure?

Figure 3 can be created from one of the hexagons by reflecting it in a vertical line, followed by a horizontal reflection (or vice versa).



How could you use translation and reflection to create this design?

WWW Web Link

To see examples of rotation symmetry, go to www.mathlinks9.ca and follow the links.

Show You Know

Consider each figure.

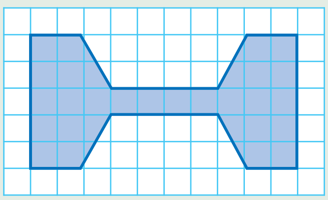


Figure A

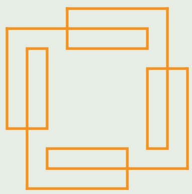


Figure B

a) Does the figure show line symmetry, rotation symmetry, or both?

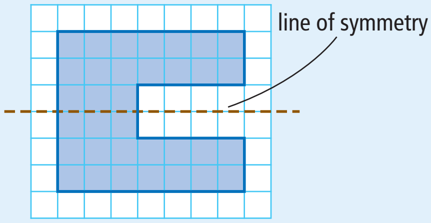
b) If the figure has line symmetry, describe each line of symmetry as vertical, horizontal, or oblique.

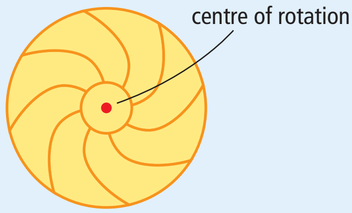
c) For each example of rotation symmetry, give the order of rotation.

d) How could each design be created from a single part of itself using translations, reflections, or rotations?

Key Ideas

- The two basic kinds of symmetry for 2-D shapes or designs are
  - line symmetry
  - rotation symmetry





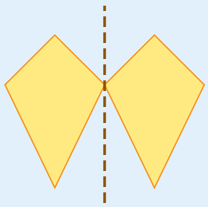
- The order of rotation is the number of times a figure fits on itself in one complete turn.

  For the fan shown above, the order of rotation is 8.

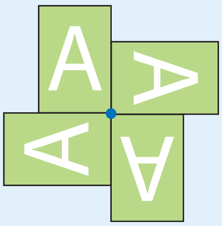
- The angle of rotation is the smallest angle through which the shape or design must be rotated to lie on itself. It is found by dividing the number of degrees in a circle by the order of rotation.

  For the fan shown above, the angle of rotation is $360° \div 8 = 45°$ or $1 \div 8 = \frac{1}{8}$, or $\frac{1}{8}$ turn.

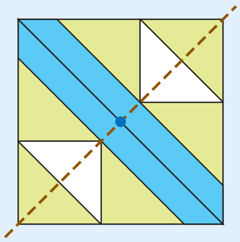
- A shape or design can have one or both types of symmetry.



line symmetry



rotation symmetry



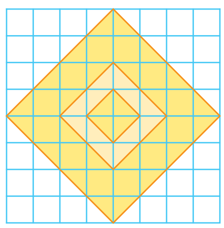
both

Check Your Understanding

Communicate the Ideas

1. Describe rotation symmetry. Use terms such as centre of rotation, order of rotation, and angle of rotation. Sketch an example.

2. Maurice claims the design shown has rotation symmetry. Claudette says that it shows line symmetry. Explain how you would settle this disagreement.



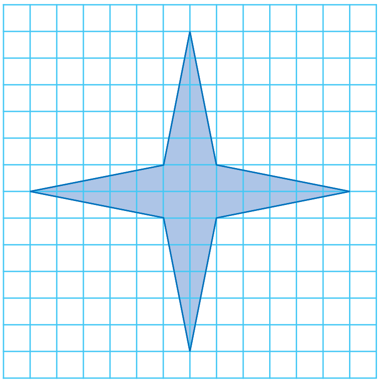
3. Can a shape and its translation image demonstrate rotation symmetry? Explain with examples drawn on a coordinate grid.

Practise

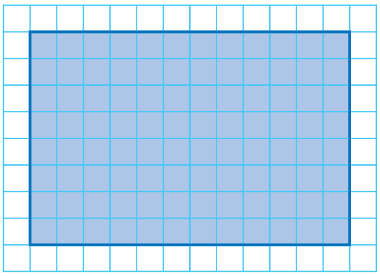
For help with #4 and #5, refer to Example 1 on page 17.

4. Each shape or design has rotation symmetry. What is the order and the angle of rotation? Express the angle in degrees and as a fraction of a turn. Where is the centre of rotation?

a)



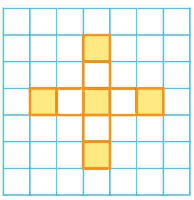
b)



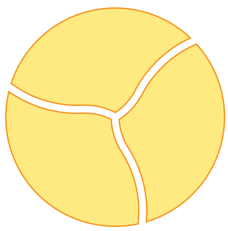
c) 1961

5. Does each figure have rotation symmetry? Confirm your answer using tracing paper. What is the angle of rotation in degrees?

a)



b)

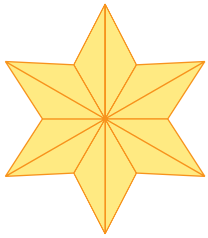


c) XOX

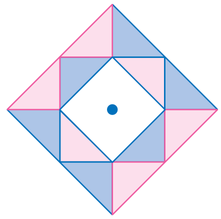
For help with #6 and #7, refer to Example 2 on pages 18–19.

6. Each design has line and rotation symmetry. What are the number of lines of symmetry and the order of rotation for each?

a)



b)



c)

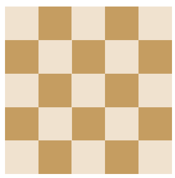
7. Each design has both line and rotation symmetry. Give the number of lines of symmetry and the size of the angle of rotation for each.

a)

b)

Apply

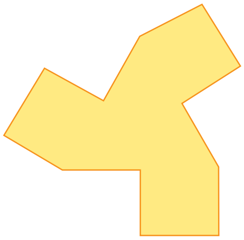
8. Examine the design.



a) What basic shape could you use to make this design?

b) Describe how you could use translations, rotations, and/or reflections to create the first two rows of the design.

9. Consider the figure shown.



a) What is its order of rotation?

b) Trace the figure onto a piece of paper. How could you create this design using a number of squares and triangles?

c) Is it possible to make this figure by transforming only one piece? Explain.

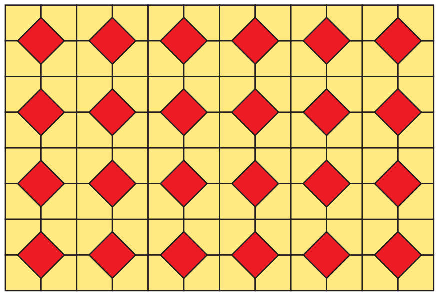
10. Many Aboriginal languages use symbols for sounds and words. A portion of a Cree syllabics chart is shown.

ᐁ e		ᐃ i	ᐄ ii	ᐅ u	ᐆ uu	ᐊ a	ᐋ aa	
ᐍ we			ᐑ wii			ᐘ wa	ᐚ waa	
ᐯ pe	ᐻ	ᐱ pi	ᐲ pii	ᐳ pu	ᐴ puu	ᐸ pa	ᐹ paa	ᑆ pwaa
ᑌ te	ᑘ twe	ᑎ ti	ᑏ tii	ᑐ tu	ᑑ tuu	ᑕ ta	ᑖ taa	ᑤ twaa
ᑫ ke	ᑵ kwe	ᑭ ki	ᑮ kii	ᑯ ku	ᑰ kuu	ᑲ ka	ᑳ kaa	ᒁ kwaa

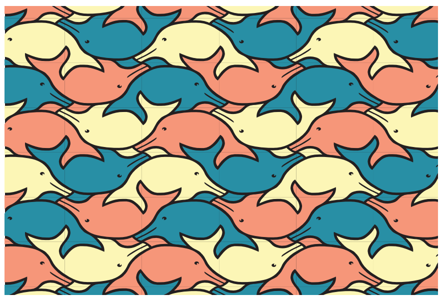
a) Select two symbols that have line symmetry and another two that have rotation symmetry. Redraw the symbols. Show the possible lines of symmetry and angles of rotation.

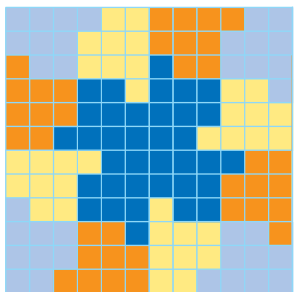
b) Most cultures have signs and symbols with particular meaning. Select a culture. Find or draw pictures of at least two symbols from the culture that demonstrate line symmetry or rotation symmetry. Describe what each symbol represents and the symmetries involved.

11. Does each tessellation have line symmetry, rotation symmetry, both, or neither? Explain by describing the line of symmetry and/or the centre of rotation. If there is no symmetry, describe what changes would make the image symmetrical.

a) 

b) 

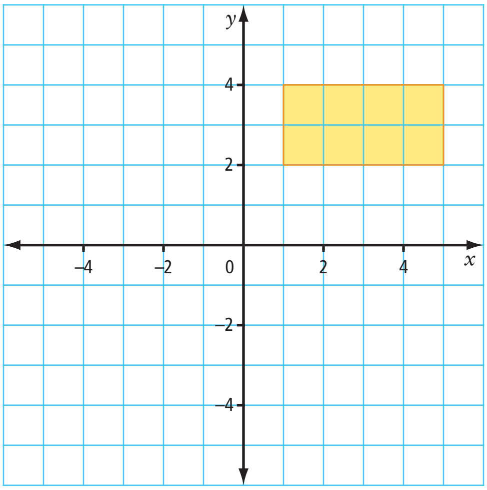
c) 

d) 

Literacy Link

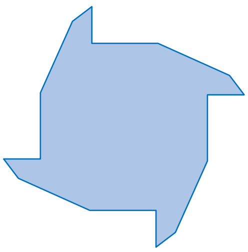
A *tessellation* is a pattern or arrangement that covers an area without overlapping or leaving gaps. It is also known as a tiling pattern.

12. Reproduce the rectangle on a coordinate grid.
   a) Create a drawing that has rotation symmetry of order 4 about the origin. Label the vertices of your original rectangle. Show the coordinates of the image after each rotation.



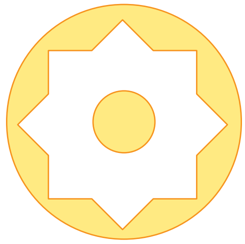
   b) Start again, this time using line symmetry to make a new design. Use the y-axis and then the x-axis as a line of symmetry. How is this new design different from the one that you created in part a)?

13. Sandra makes jewellery. She created a pendant based on the shape shown.



   a) Determine the order and the angle of rotation for this design.
   b) If Sandra's goal was to create a design with more than one type of symmetry, was she successful? Explain.

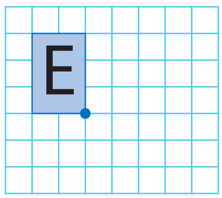
14. Alain drew a pendant design that has both line and rotation symmetry.



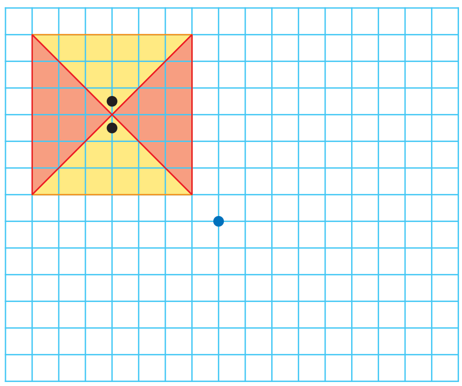
   a) How many lines of symmetry are in this design? What is the size of the smallest angle between these lines of symmetry?
   b) What are the order and the angle of rotation for this design?

15. Imagine you are a jewellery designer. On grid paper, create a design for a pendant that has more than one type of symmetry. Compare your design with those of your classmates.

16. Copy and complete each design. Use the centre of rotation marked and the order of rotation symmetry given for each part.
   a)



   Order of rotation: 2
   b)



   Order of rotation: 4
   Hint: Pay attention to the two dots in the centre of the original shape.

17. Automobile hubcaps have rotation symmetry. For each hubcap shown, find the order and the angle of rotation in degrees.

a)

b)

c)

d)

18. a) Sometimes the order of rotation can vary depending on which part of a diagram you are looking at. Explain this statement using the diagram below.

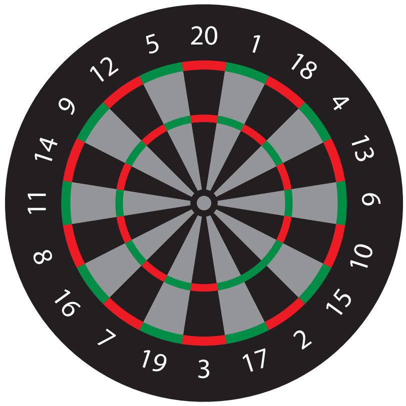
b) How would you modify this diagram so that it has rotation symmetry?

19. a) Describe the symmetry shown on this playing card.

b) Why do you think the card is designed like this?

c) Does this playing card have line symmetry? Explain.

20. Two students are looking at a dart board. Rachelle claims that if you ignore the numbers, the board has rotation symmetry of order 10. Mike says it is order 20. Who is correct? Explain.

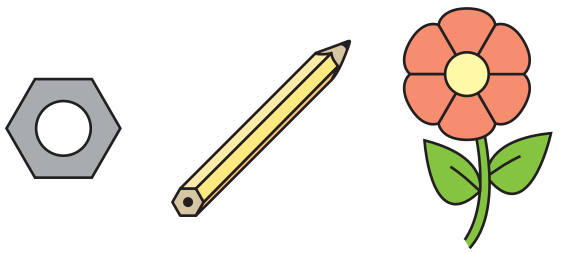


21. a) Which upper-case letters can be written to have rotation symmetry?

b) Which single digits can be considered to have rotation symmetry? Explain your answer.

c) Create a five-character Personal Identification Number (PIN) using letters and digits that have rotational symmetry. In addition, your PIN must show line symmetry when written both horizontally and vertically.

22. Some part of each of the objects shown has rotation symmetry of order 6. Find or draw other objects that have rotation symmetry of order 6. Compare your answers with those of some of your classmates.



23. Organizations achieve brand recognition using logos. Logos often use symmetry.

a) For each logo shown, identify aspects of symmetry. Identify the type of symmetry and describe its characteristics.

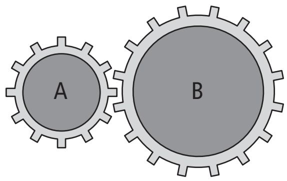
A

B

b) Find other logos that have line symmetry, rotation symmetry, or both. Use pictures or drawings to clearly show the symmetry involved.

Extend

24. Two gears are attached as shown.



a) The smaller gear has rotation symmetry of order m. What is the value of m? What could m represent?

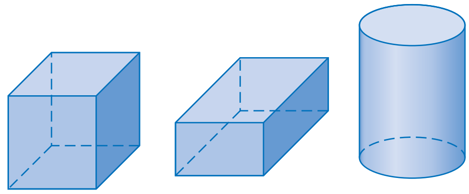
b) The larger gear has rotation symmetry of order n. Find the value of n.

c) When the smaller gear makes six full turns, how many turns does the larger gear make?

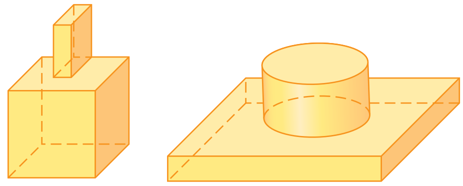
d) If gear A has 12 teeth, and gear B has 16 teeth, how many turns does B make when A makes 8 turns?

e) If gear A has x teeth, and gear B has y teeth, how many turns does B make when A makes m turns?

25. Examine models or consider these drawings of the 3-D solids shown.



Group A



Group B

a) Select one object from each group. Discuss with a partner any symmetry that your selected objects have.

b) For one of the objects you selected, describe some of its symmetries. Use appropriate mathematical terminology from earlier studies of solids and symmetry.

26. A circle has a radius of length r. If a chord with length r is rotated about the centre of the circle by touching end to end, what is the order of rotation of the resulting shape? Explain.

Math Link

Your design company continues to expand. As a designer, you are constantly trying to keep your ideas fresh. You also want to provide a level of sophistication not offered by your competitors. Create another appealing design based on the concepts of symmetry you learned in section 1.2. Sketch your design on a half sheet of 8.5×11 paper. Store it in the pocket in your Foldable. You will need this design as part of Math Link: Wrap It Up! on page 39.