Line Symmetry

After this lesson, you will be able to…

- **classify 2-D shapes or designs according to the number of lines of symmetry**
- **identify the line(s) of symmetry for a 2-D shape or design**
- **complete a shape or design given one half of the shape and a line of symmetry**
- **create a design that demonstrates line symmetry**

Materials

- scissors
- isometric dot paper
- tracing paper
- grid paper

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line of symmetry

- a line that divides a figure into two reflected parts
- sometimes called a line of reflection or axis of symmetry
- a figure may have one or more lines of symmetry, or it may have none
- can be vertical, horizontal, or oblique (slanted)

This optical illusion was developed around 1915 by the Dutch psychologist Edgar Rubin. Like many optical illusions, this one involves a reflection. Look at the illusion. What do you see? Where is the line of reflection in this example?
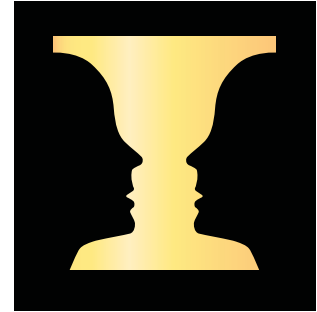
Explore Lines of Symmetry

1. Fold a piece of paper in half. Mark two points, A and B, on the fold. Draw a wavy or jagged line between points A and B on one side of the paper. Cut along the line and then unfold your cut-out figure.

a) How does the fold affect the shape of the cutout?

b) Explain why it would make sense to refer to your fold line as a **line of symmetry**.
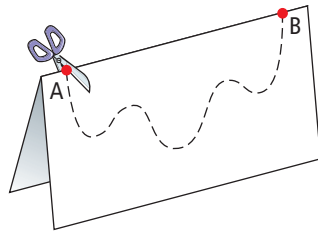
2. a) How could you fold a piece of paper so that a cutout shape would have two lines of symmetry? Use your method to create a cutout with two lines of symmetry.

b) Fold and cut a piece of paper to make a design with four lines of symmetry.

3. Draw an equilateral triangle on isometric dot paper. Then, cut it out using scissors. How many lines of symmetry are there? Explain how you arrived at your answer.

4. The diagram shows half of a shape. Line *r* represents a line of symmetry for the shape. Copy the diagram. Then, draw the complete shape.
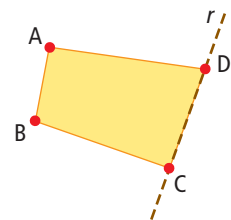
Reflect and Check

5. What are some ways to complete the shape in #4? Describe one way to a partner. See if your partner can follow your instructions.

6. Describe two different ways to find a line of symmetry for a symmetric 2-D shape. Which one do you prefer? Why?

Link the Ideas

Symmetry creates a sense of balance and, often, a sense of peace, tranquility, and perfection. A famous example of its use in architecture is the Taj Mahal, in Agra, India. Many parts of the building and grounds were designed and built to be perfectly symmetrical. Symmetry can also be seen in the pools that reflect an image of the structure. However, as with most cases of naturally occurring symmetry, the reflection in the pools is not perfect.

Example 1: Find Lines of Symmetry

Each of the following demonstrates **line symmetry**. For each part, use a different method to find the line(s) of symmetry. State the number of lines of symmetry and describe each one.

a)

b)

c)
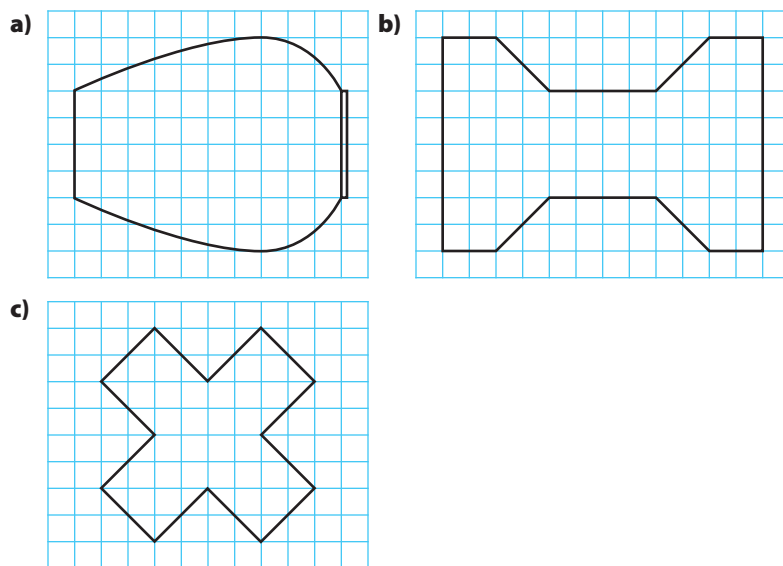
line symmetry

- a type of symmetry where an image or object can be divided into two identical, reflected halves by a line of symmetry
- identical halves can be reflected in a vertical, horizontal, or oblique (slanted) line of symmetry
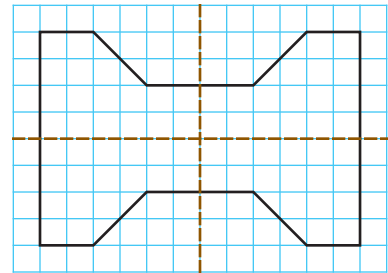
vertical line of symmetry

horizontal line of symmetry

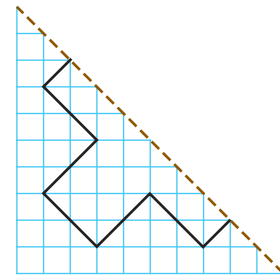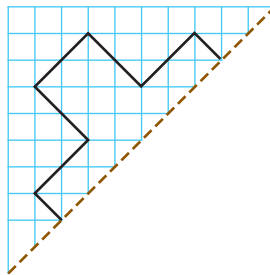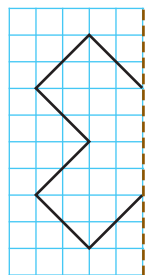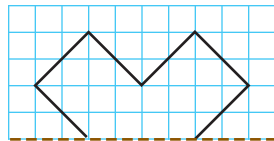oblique line of symmetry
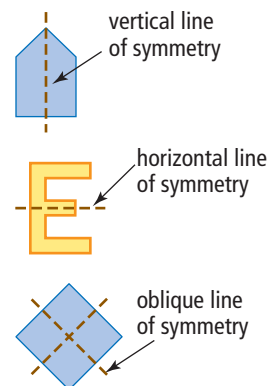
Solution

a) By using a Mira™, you can see that there is one horizontal line of symmetry.
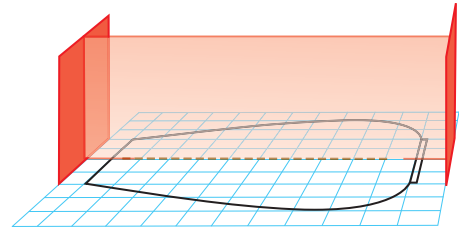
b) You can find the lines of symmetry by counting on the grid. For this figure, there are the same number of squares above and below the horizontal line of symmetry. There are the same the number of squares to the left and right of the vertical line of symmetry. You can see that there are two lines of symmetry: one horizontal and one vertical.


c) You can find the lines of symmetry by folding. If the shape on each side of the fold is the same, the fold line is a line of symmetry. This figure can be folded along four different lines to create mirrored shapes: one horizontal, one vertical, and two oblique.

You can sketch the complete figures to prove that the fold lines in these images are, in fact, lines of symmetry?

Show You Know

How many lines of symmetry are possible for each figure? Describe each line of symmetry as vertical, horizontal, or oblique.

**a)** **b)** **c)**
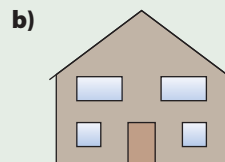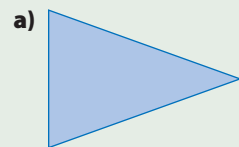
Example 2: Complete Drawings Using Symmetry

Each drawing shows half of a figure. The dashed brown line represents a line of symmetry for the figure. Draw a complete version of each figure.

a)

b)

Solution

a) *Method 1: Use Paper Folding*

Fold a piece of paper in half. Draw the figure on the paper so that the line of symmetry is along the folded edge. Cut out the figure you have drawn. Unfold the paper to reveal the complete figure.
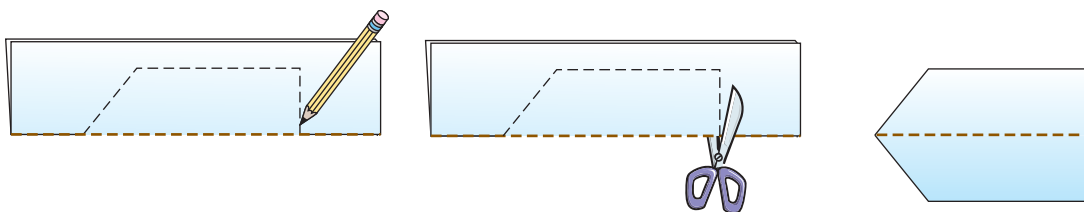
Method 2: Use Measurement or Counting

Draw the half figure onto a grid, and label the vertices A, B, C, and D. All points not on the line of symmetry are reflected on the opposite side of the line. In this figure, this is points B and C. The reflected points are drawn the same perpendicular distance from the fold line so that BX = B′X and CD = C′D. Join A to B′, B′ to C′, and C′ to D to complete the figure.
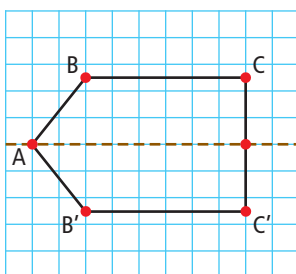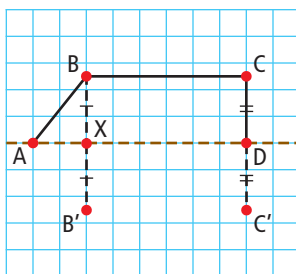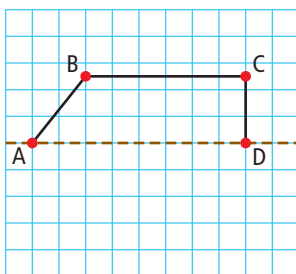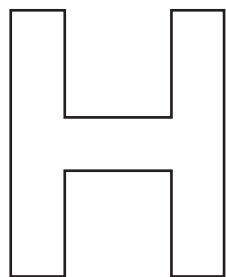
b) One method is to mark the perpendicular distance from each vertex on the opposite side of the line of symmetry. Then, connect the lines to complete the figure.
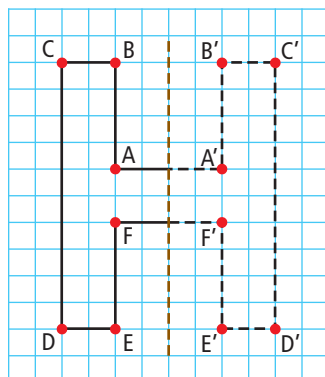
What is another method that you could use to create this figure?

The completed figure is a block-letter H.
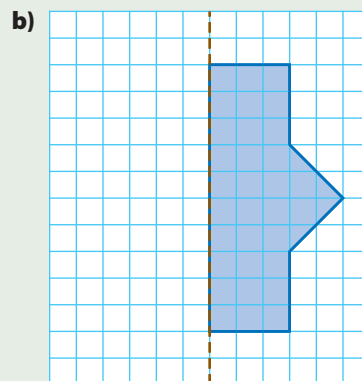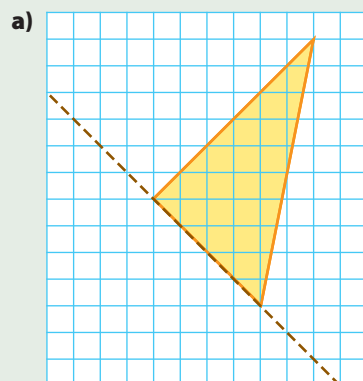Notice that the line of symmetry is not part of the final figure.

Show You Know

Copy each shape. Use the line of symmetry and a method of your choice to complete each shape.
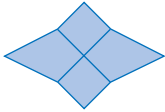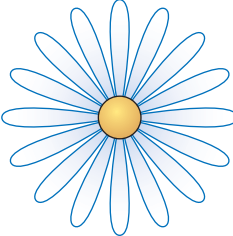
a)

b)

Key Ideas

- Line symmetry exists whenever a shape or design can be separated into two identical halves by a line of symmetry. The line of symmetry, also known as a line of reflection, may or may not be part of the diagram itself.
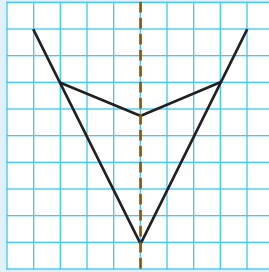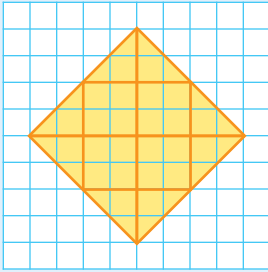
- A shape or design can have any whole number of lines of symmetry.

Shape			
Number of Lines of Symmetry	0	2	16

Describe the lines of symmetry in these images.

- You can complete a symmetric drawing by folding or reflecting one half in the line of symmetry. The opposite halves are mirror images.

O T | T O

This name has one line of symmetry. If you know the first two letters you can complete the name by reflecting in the dashed line.

Literacy Link

If a shape or design has symmetry, then it can be described as *symmetric* or *symmetrical*.

Check Your Understanding

Communicate the Ideas

1. Any rectangle has only two lines of symmetry. Do you agree or disagree with this statement? Explain. Use drawings to support your argument.

2. Explain the changes you would need to make in the diagram so that the diagonal lines in the centre would become lines of symmetry. Redraw the diagram to match your answer.
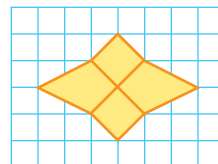
3. Three students disagree on whether a parallelogram is symmetric. Sasha claims it is symmetric and has two lines of symmetry. Basil says it is symmetric and has four lines of symmetry. Kendra argues it is not symmetric since it has no lines of symmetry. Which of the three answers is correct? Explain why.
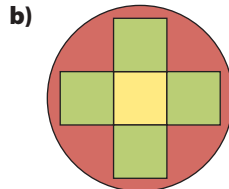
> **Literacy Link**
>
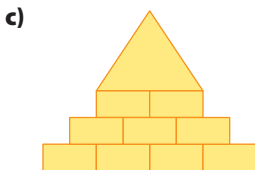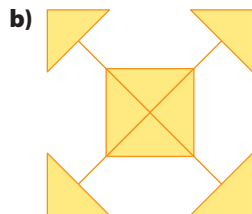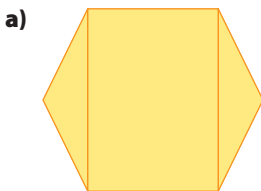> A *parallelogram* is a four-sided figure with opposite sides parallel and equal in length.
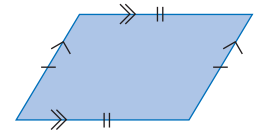
Practise

For help with #4 to #6, refer to Example 1 on pages 7–8.

4. Where are the lines of symmetry for each figure? Draw a rough sketch of the figures in your notebook. Show all lines of symmetry in a different colour.

a)

b)

c)

5. Redraw each diagram, showing all lines of symmetry.

a)

b)

c)

6. Which figures have only two lines of symmetry? Explain how you know.

A

B
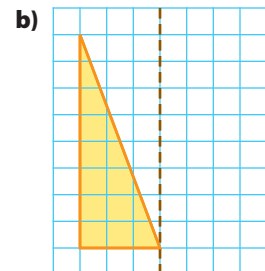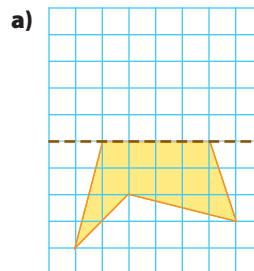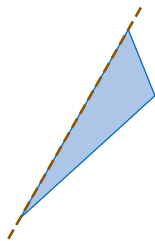
C

D

E

F
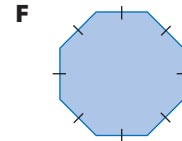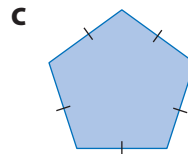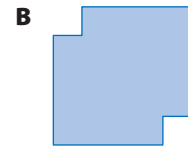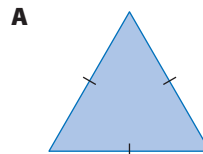
For help with #7 and #8, refer to Example 2 on pages 9–10.

7. If the dashed line is the line of symmetry, what does the complete diagram look like? Sketch your diagrams on grid paper.

a)

b)

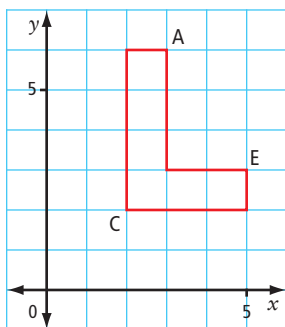**8.** Copy each figure. Use the line of symmetry shown to complete each figure.

a)

b)

Apply

9. Copy the figure on a coordinate grid.

a) Draw the reflection image if the *y*-axis is the line of reflection. Label the reflected vertices A′, C′, and E′.

b) What are the coordinates of A′, C′, and E′ in your drawing in part a)?

c) Do the original figure and its reflection image show line symmetry? Explain.

10. Create a figure similar to question #9, using a coordinate grid.

a) Translate the figure 4 units to the right.

b) What are the coordinates of A′, C′, and E′?

c) Do the original figure and its translation image show line symmetry? Explain your thinking.

d) Now, translate the figure you created in part a) 5 units down. Do the original figure and this new translation image show line symmetry? Explain.

11. Some regular shapes, such as an equilateral triangle, a square, or a regular hexagon, appear to show line symmetry when they are translated in one direction. Do you agree or disagree with this statement? Give examples to support your argument. Discuss your answer with a partner.

12. The Norwegian flag has a width to length ratio of 8 to 11.

a) Does the flag have line symmetry? Explain your answer.

b) What changes would be necessary in order to have exactly two lines of symmetry?

13. How many lines of symmetry does the flag of each of the following countries have?

a) Belgium

b) Canada

c) Scotland

d) Switzerland

Did You Know?

There are only two sovereign states that have a square flag: Switzerland and Vatican City. The flag of Belgium is close to square, with a width to length ratio of 13:15.

14. The number of lines of symmetry for a square flag can vary. Create sketches of flag designs that show 0, 1, 2, and 4 lines of symmetry.

15. Consider the upper-case block letters of the English alphabet.

A B C D E F G H I J K L M N
O P Q R S T U V W X Y Z

a) Which letters have a horizontal line of symmetry?

b) Which letters have a vertical line of symmetry?

**c)** Which letter(s) have both horizontal and vertical lines of symmetry?

16. Using block letters, the word MOM can be written either vertically or horizontally. In each position, it has one vertical line of symmetry.
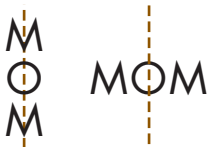
a) Write at least two other words that have one vertical line of symmetry when printed vertically or horizontally.

b) Find a word that has a horizontal line of symmetry when it is printed horizontally, and a vertical line of symmetry when printed vertically.

c) Find a word that has one line of symmetry when it is printed vertically, but that is not symmetric when printed horizontally.

17. a) Some single digits have line symmetry, depending on how they are printed. Which digits can demonstrate line symmetry?

b) Write a four-digit number that has two lines of symmetry when written horizontally.

c) What is a five-digit number that has two lines of symmetry?

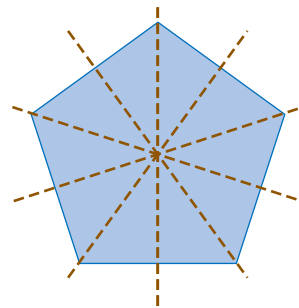**18.** Margaux is exploring regular polygons and line symmetry. She discovers that

- an equilateral triangle has three interior angles and three lines of symmetry
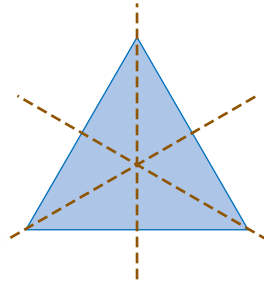
- a square has four interior angles and four lines of symmetry
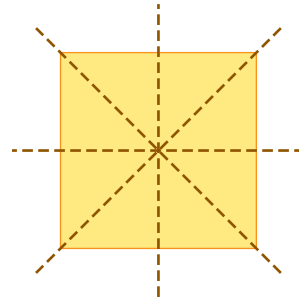
- a regular pentagon has five interior angles and five lines of symmetry

a) Work with a partner to continue Margaux's exploration for a regular hexagon, heptagon, and octagon.

b) What pattern do you discover?

c) Does this pattern continue beyond an octagon? How do you know?
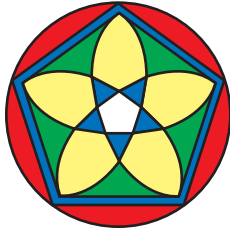
19. Consider these figures.
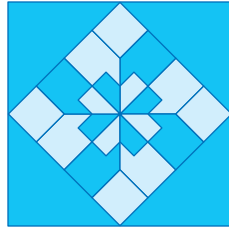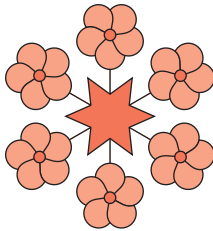

Figure A

Figure B

Figure C

a) Which figure(s) shows line symmetry?
b) What effect does the colour have on your answer in part a)?
c) How many lines of symmetry does each figure you identified in part a) have?

Extend

20. The triangle A(−6, 0) B(−2, 0) C(−2, −3) is first reflected in the *y*-axis. The resulting triangle is then reflected in a vertical line passing through (10, 0) to form △A″B″C″. Describe one transformation that translates △ABC directly to △A″B″C″.

21. Consider the clocks shown.

A

B
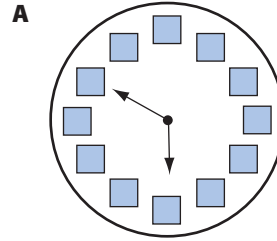
a) Explain whether each clock has line symmetry at some time? Ignore the different lengths of the hands on clock A.
b) At what time(s) do your choices in part a) show true line symmetry?

22. Points A(4, 3) and B(6, −4) are reflected in the *y*-axis to form a quadrilateral. What is its area?

23. The triangle A(−6, 0) B(−2, 0) C(−2, −3) is reflected in the *y*-axis. The resulting image is then reflected in a diagonal line passing through the origin and (5, 5) to form △A″B″C″. Describe one transformation that translates △ABC directly to △A″B″C″.

24. A three-dimensional object that is cut in half by a plane may be symmetric. Do you agree? Give examples.

> **◐◑ Literacy Link**
>
> A *plane* is a flat, two-dimensional surface that extends in all directions.

Math Link

Imagine that you are working for a company that produces designs for many different uses, from playing cards to novelty items. Your job is to create appealing designs that can be used for a variety of products. As part of your portfolio, create a design that has at least two lines of symmetry. Draw your design on a half sheet of 8.5 × 11 paper. Store it in the pocket in your Foldable. You will need this design for the Math Link: Wrap It Up! on page 39.